Post-Quantum Security of the Fujisaki-Okamoto (FO) and OAEP Transforms

Made by:
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Joint work with **Dominique Unruh**

Motivation: Post-Quantum Cryptography

Users intend to use classical cryptographic schemes, however, the adversary has the **quantum computing power**.

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   - E.g. Security proofs in the Random Oracle Model.
     - Relative to a specific oracle, the Fiat-Shamir transform is insecure in the quantum setting. [1]

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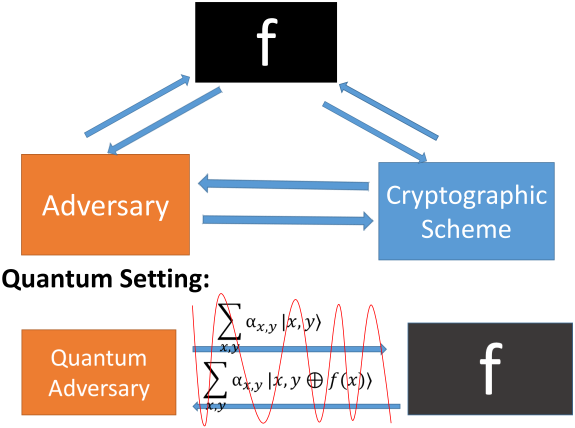
Random Oracle Model in quantum setting



[2] [Boneh et al. Random Oracles in a Quantum World. ASIACRYPT 2011]

Fujisaki-Okamoto (FO) transform

$$Enc^{hy}_{pk}(m;\delta) = \left(Enc^{asy}_{pk}\left(\delta; H\left(\delta \| Enc^{sy}_{G(\delta)}(m)\right)\right),\ Enc^{sy}_{G(\delta)}(m)\right)$$

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Properties of encryption schemes:

- The symmetric encryption scheme is One-time secure.
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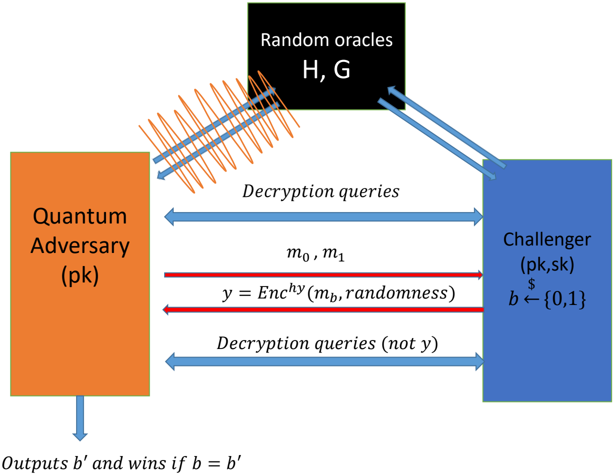
Question: What about security in the Quantum Random Oracle Model (QROM)?

Our contribution

Security of the slightly modified Fujisaki-Okamoto and OAEP transforms in the **Quantum** Random Oracle Model.

IND-CCA in the QROM



Challenges in the Quantum setting

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Solutions to the Challenges

1. List of $(x, H(x))$ and $(x, G(x))$ are needed!
   - Add $H'(\delta)$ to the ciphertext $\left(Enc^{asy}_{pk}\left(\delta; H\left(\delta \| Enc^{sy}_{G(\delta)}(m)\right)\right),\ Enc^{sy}_{G(\delta)}(m), H'(\delta)\right)$.

[3] [Unruh, Revocable quantum timed-release encryption, Eurocrypt 2014]
[4] [Targhi, Tabia, Unruh. Quantum Collision-Resistance of Non-uniformly Distributed Functions. PQCrypto 2016]

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   - Using "One-way to hiding" Lemmas[3] as a tool to reprogramme the random oracle

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Comment: The same proof techniques work for OAEP transform

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Question?

Thank you for listening!