Shannon Effect for BC-complexity of Finite Automata

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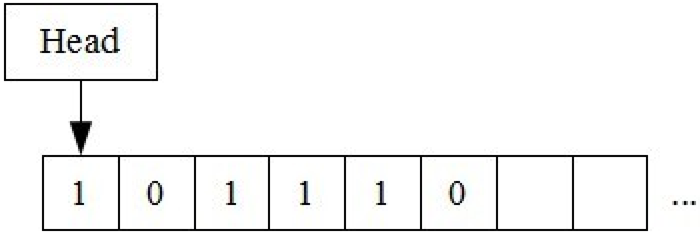
2014.10.04

Outline of the talk

- Finite automata (DFA)
- Representation of automata with Boolean circuits
- BC-complexity
- Shannon effect for BC-complexity
- NFA, language operations
- Minimization

A finite automaton (DFA) consists of:



- Input tape
- Read-only head moving in only one direction
- On each step
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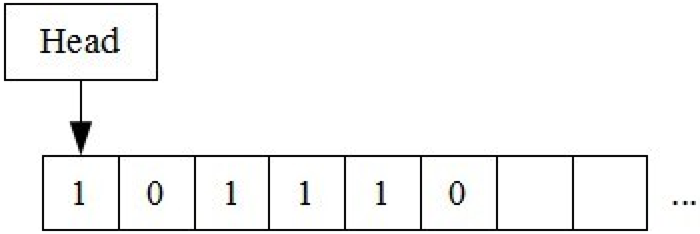

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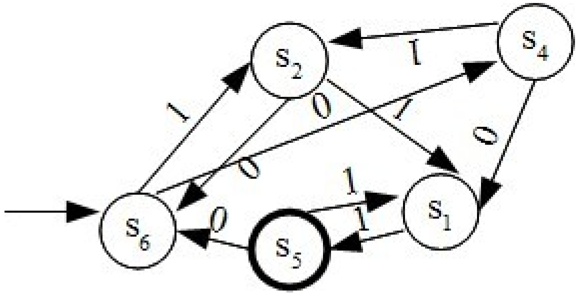

Finite Automata

A deterministic finite automaton (DFA) is a tuple $A(\Sigma, Q, Q_F, \delta, q_0)$ where

- Σ is the input alphabet
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What is the most complex automaton that we can build (model on a computer)?

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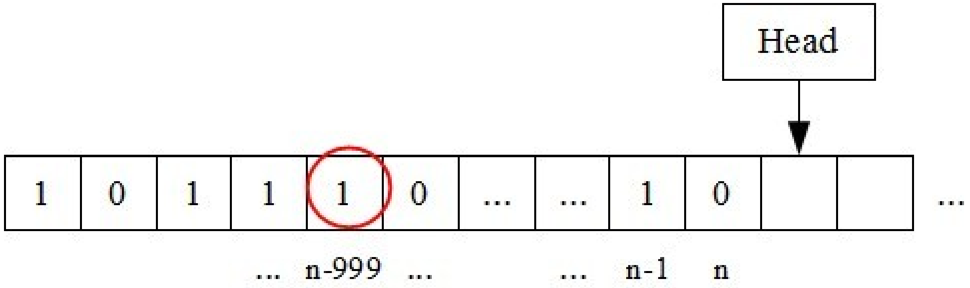
- 2^{10} states?
- 2^{100} states?
- 2^{1000} states?

Automata with 2^{1000} states

1. Automaton A_1 accepts language L_1 of words in a binary alphabet $\Sigma=\{0, 1\}$ for which 1000th digit from the end is "1".

$$\mathbf{x} \in L_1 \Leftrightarrow x_{|x|-999}=1$$



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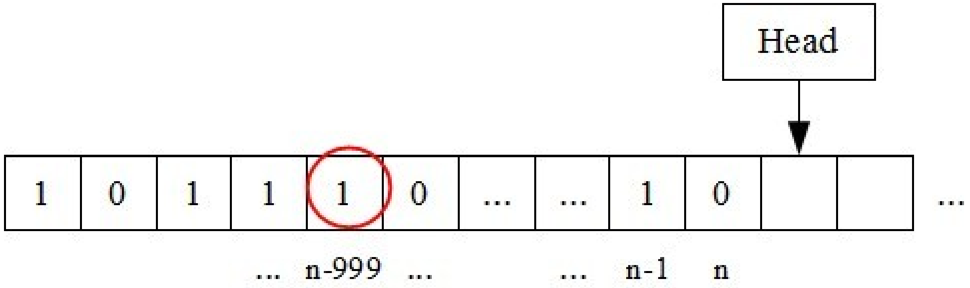
- Implementation – use 1000 bit LIFO register



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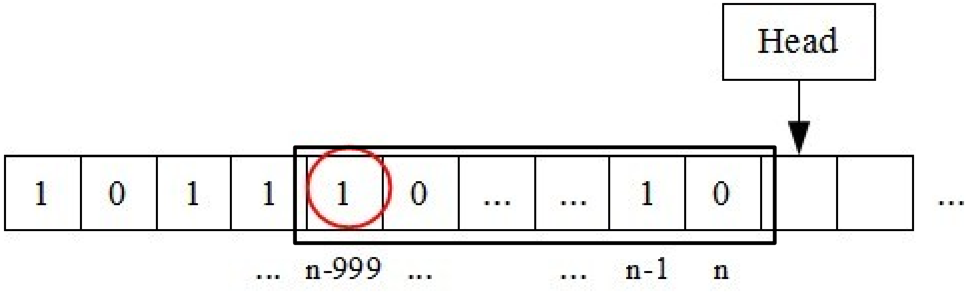

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- Transition function $\delta : \Sigma \times Q \to Q$ → Boolean circuit:
  - *Inputs* : *input bits and state bits*
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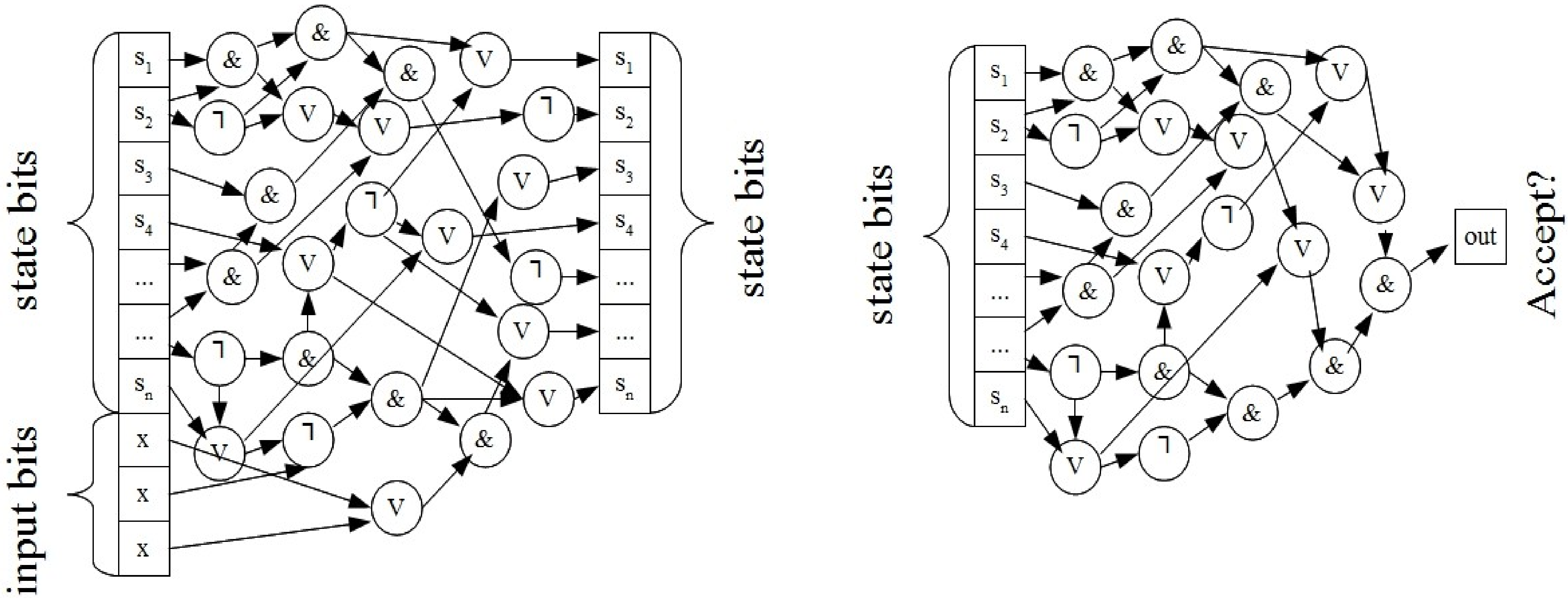
- Set of final states $Q_F \subseteq Q$ → a Boolean circuit for the characteristic function of the set Q_F:
  - *Inputs* : *state bits*
  - *Outputs* : *one bit (accept/reject)*

Representation of an automaton with two Boolean circuits:



≥log|Q| state bits ≥log|Σ| input bits

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- Each automaton can have infinitely many encodings
- Each encoding can have infinitely many representations
- (Number of state bits) $b_Q \geq \log_2(|Q|)$
- Two automata may have the same representation only if they are equivalent.

BC-complexity

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BC-complexity of a regular language

$C_{BC}(R)=\min\{C_{BC}(A): A \text{ recognizes } R\}$

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Automaton A_1 accepts language L_1 of words for which the *n* -th digit from the end is “1”:

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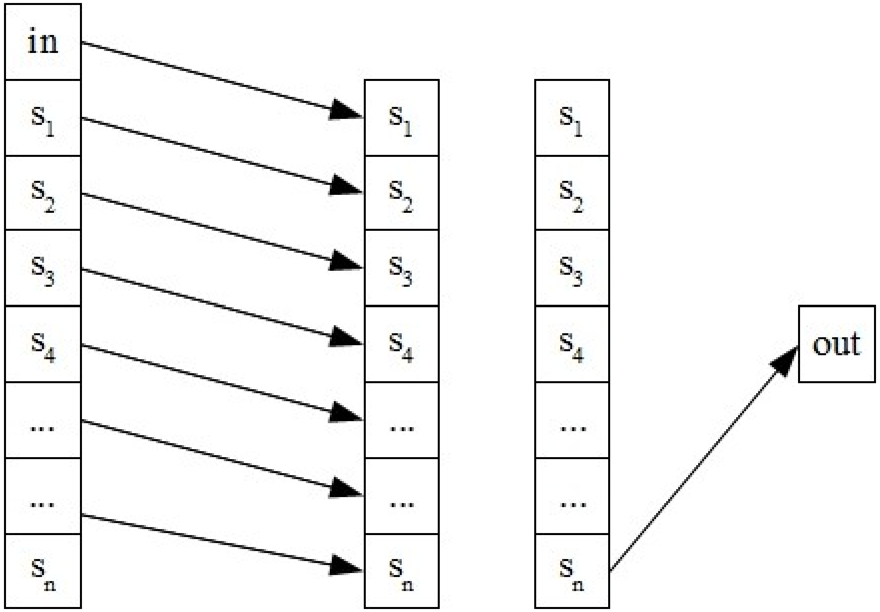

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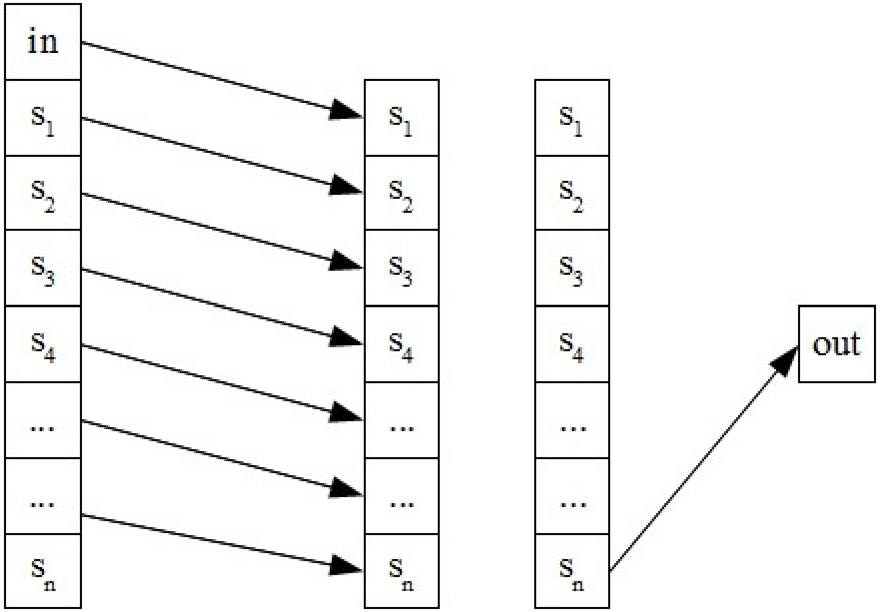

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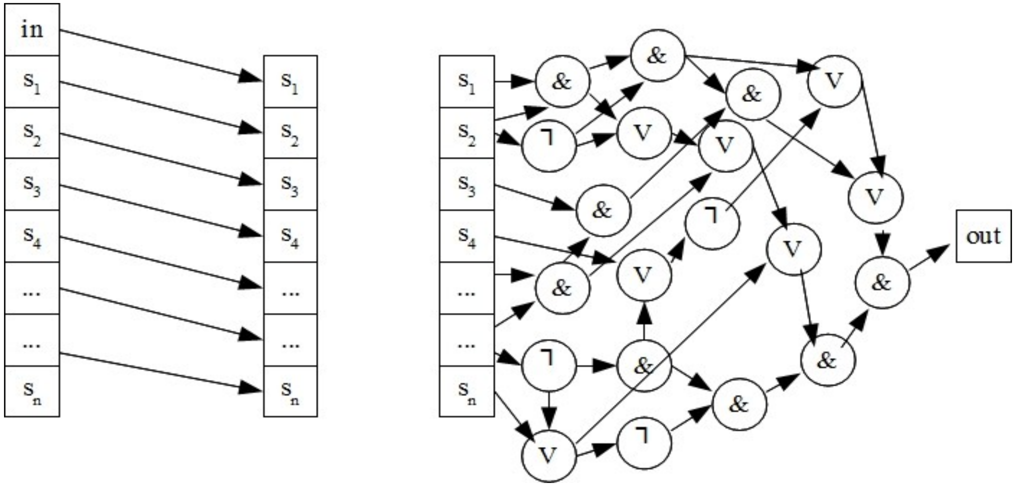

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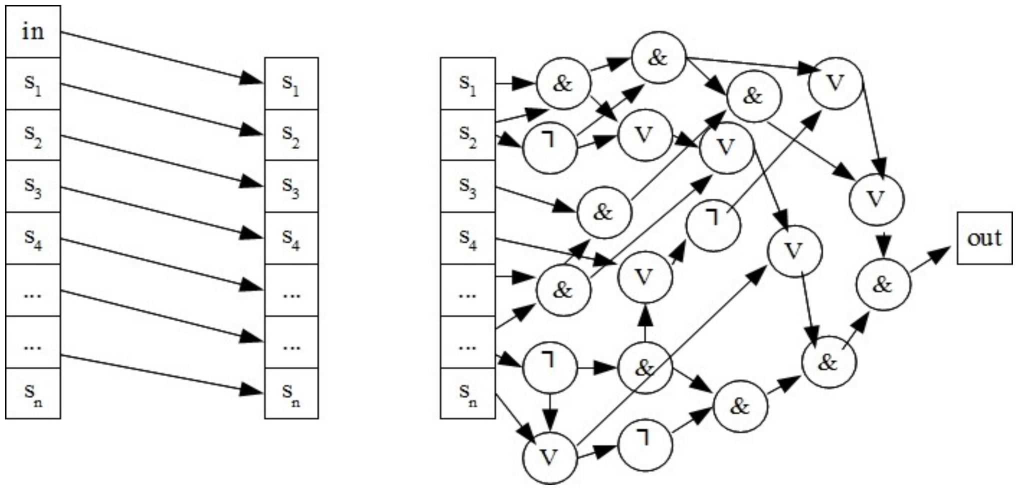

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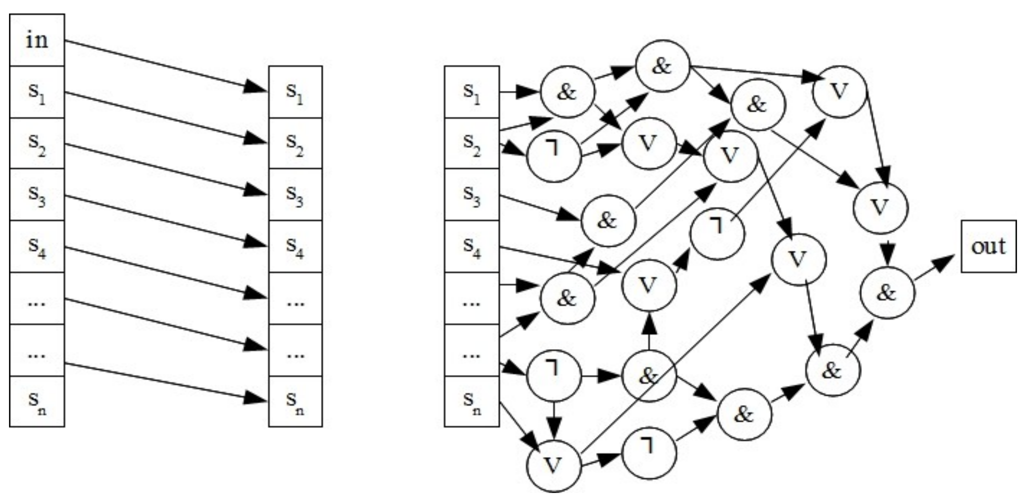

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$x \gtrsim f(n) \Leftrightarrow x > f(n)(1-o(1))$

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- $C_{BC}(A_1^n) = n$
- $C_{BC}(A_2^n) \geq 2^n/n^2$

Some special cases

- Nondeterministic automata
- Language operations

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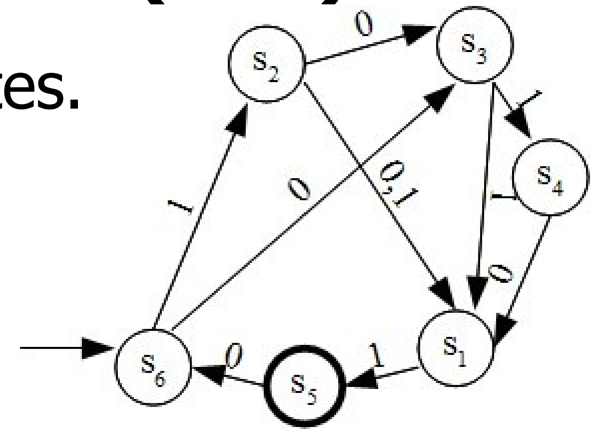

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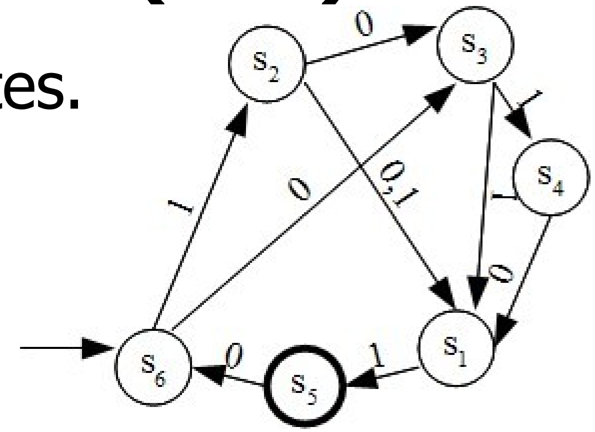

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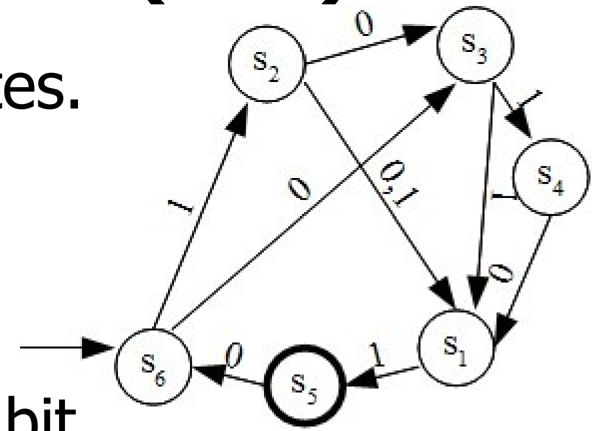

- Encode one state of NFA as one state bit.

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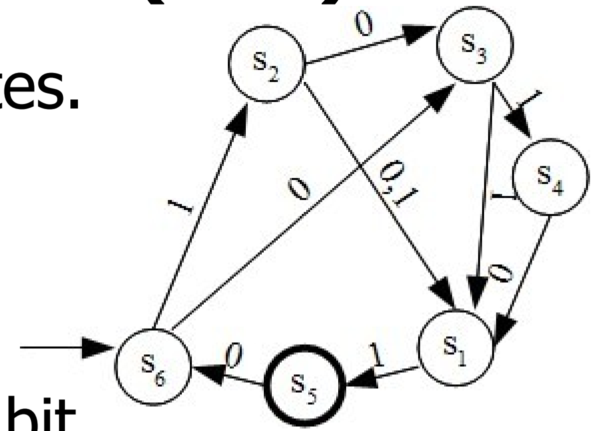

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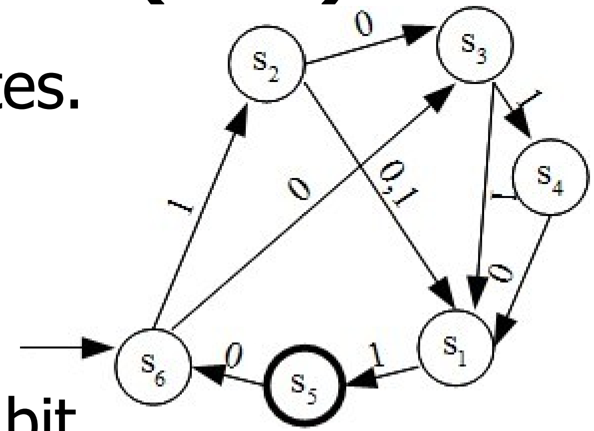

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Much simpler than a general function on *n* arguments!

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No Shannon effect right now for NFA (but coming close)

Upper bounds of BC-complexity for Language operations

- Given two languages L_1 and L_2 with state complexities *m* and *n* and BC-complexities *a* and *b*.

Operation	State comp.	BC-complexity
$L_1 \cup L_2$	mn	a+b+1
$L_1 \cap L_2$	mn	a+b+1
Σ^*-L_1	m	a+1
L_1^R	2^m	2m(m+1)
L_1L_2	$(2m-1)2^{n-1}$	2a+2n(n+1)
L_1^*	$2^{m-1}+2^{m-2}$	2m(m+1)

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- Is computer (an efficient) representation of a finite automaton?

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  - Is the BC-complexity of M(*A*) also minimal?
  - Can it be that for some automaton *A*: $C_{BC}(M(A)) \gg C_{BC}(A)$?

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If there exists a polynomial $p(x)$ such that
$C_{BC}(M(A)) < p(C_{BC}(A))$
for all automata A then PSPACE $\subseteq$ P/Poly

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For every n we can build an automaton B_n with

- Poly(n) state bits ($2^{Poly(n)}$ states)
- $C_{BC}(B_n)=Poly(n)$
- $C_{BC}(M(B_n))\notin Poly(n)$

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- We can model any automaton with a reasonable BC-complexity
- Many "naturally generated" DFAs have large state complexity but low BC complexity
- Sometimes minimizing the number of states leads to (a large) increase in BC-complexity

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- Is it always true that representation with minimal (log(|Q|) number of state bits is optimal?
- Can the upper and lower bounds for NFA be improved?
- How to estimate the lower bounds for language operations?

Open questions

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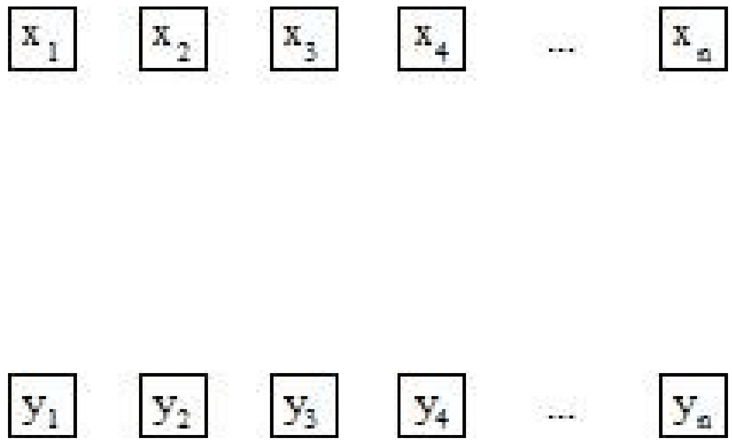

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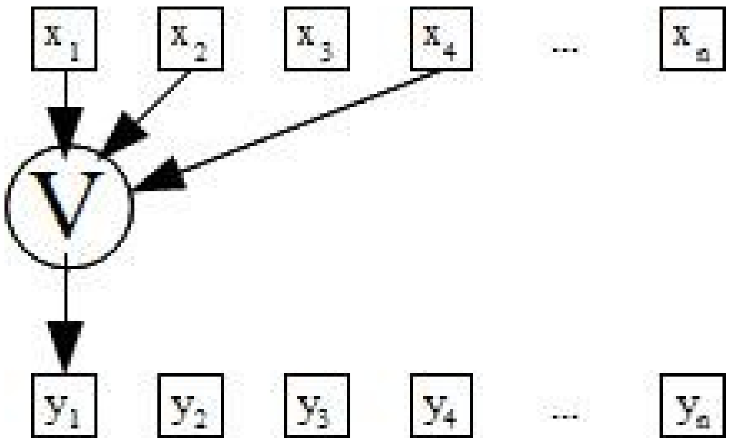

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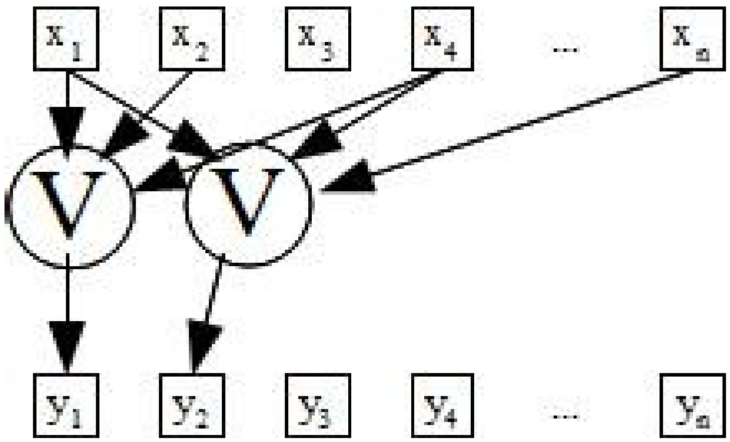

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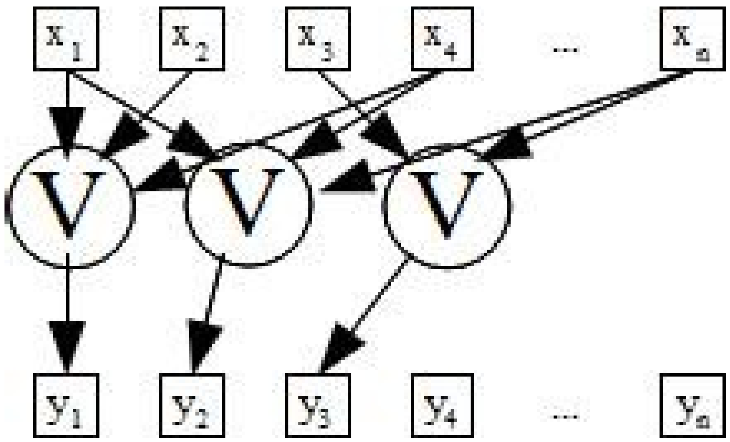

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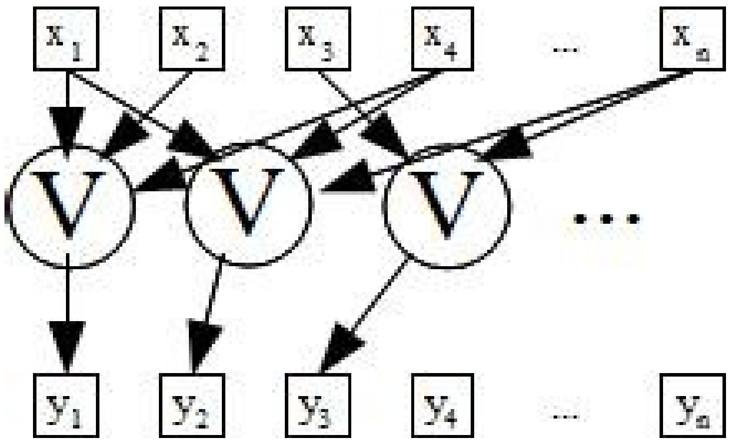

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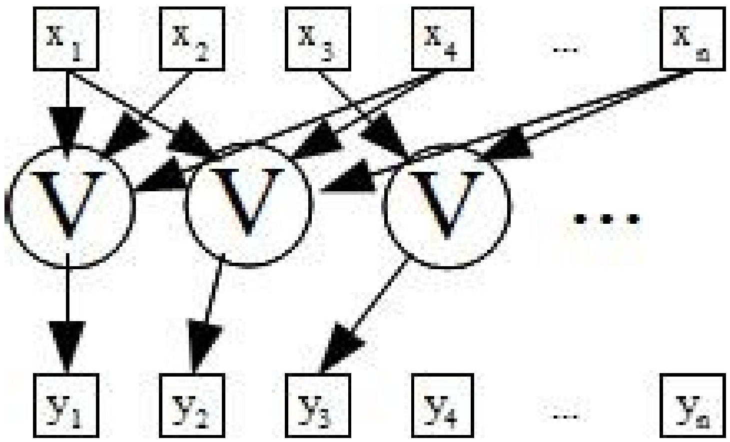

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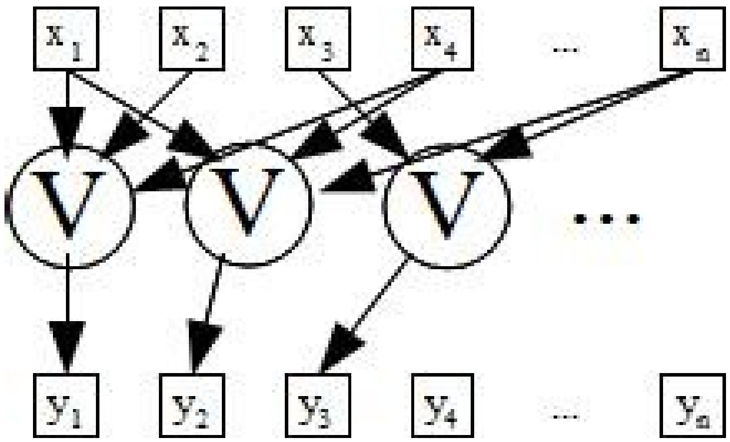

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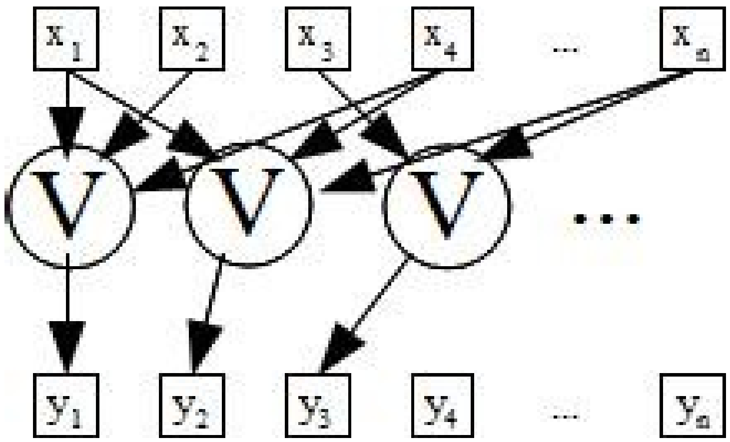

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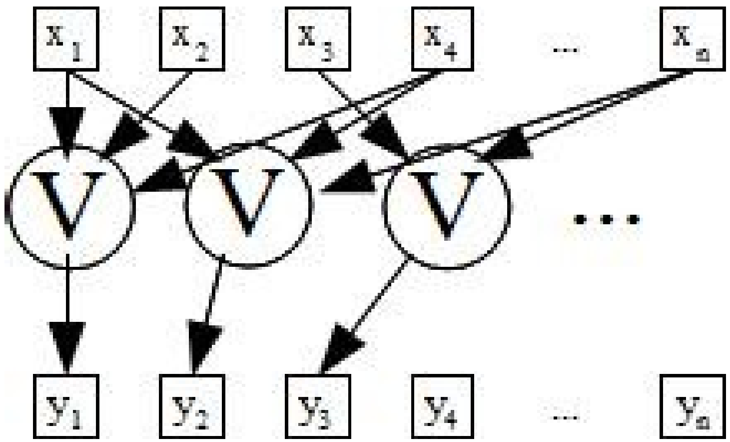

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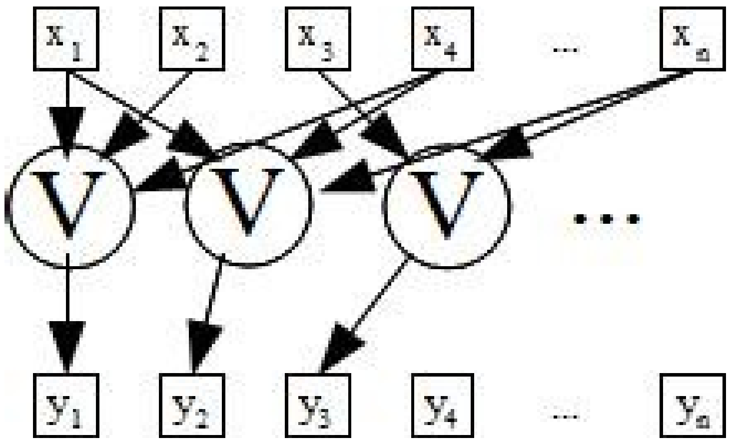

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- More efficient contruction needs asymptotically $n^2/\log n$ gates
- Can you do better?
- Lower bounds is $n^2/2\log n$ gates

Thank you!