R-trivial idempotent languages recognized by quantum finite automata

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Automata models

	"Classical" word acceptance	"Decide-and-halt" word acceptance
Deterministic Reversible Automata	Group Automata (GA) Class: Variety of group languages	Reversible Finite Automata (RFA) [Ambainis and Freivalds]
Quantum Finite Automata with pure states	Measure-Once Quantum Finite Automata (MO-QFA) [Moore et al] Class: Variety of group languages	Measure-Many Quantum Finite Automata (MM-QFA) [Kondacs and Watrous]
Probabilistic Reversible Automata	"Classical" Probabilistic Reversible Automata (C-PRA) [Golovkins and Kravtsev] Class: Variety of **BG** (block group) languages	"Decide-and-halt" Probabilistic Reversible Automata (DH-PRA) [Golovkins and Kravtsev]
Quantum Finite Automata with mixed states	Latvian Quantum Finite Automata (LQFA) [Ambainis et al, Golovkins and Kravtsev] Class: Variety of **BG** (block group) languages	Enhanced Quantum Finite Automata (EQFA) [Nayak]

Language variety

A class of recognizable languages is a function **C** that which associates with each alphabet A a set A***C** of recognizable languages of A*.

A language variety is a class of languages **C**, which is

a) closed under union, intersection and complement,
that is, for all languages L, L_1, $L_2 \in$ A***C**:
$L` \in A^*\mathbf{C}$, $L_1 \cup L_2 \in A^*\mathbf{C}$, $L_1 \cap L_2 \in A^*\mathbf{C}$;

b) closed under quotient operations,
that is, for all languages $L \in A^*\mathbf{C}$ and for all $a \in A$:
$a^{-1}L \in A^*\mathbf{C}$, $La^{-1} \in A^*\mathbf{C}$

c) closed under inverse morphisms,
that is, if φ is a morphism $A^* \rightarrow B^*$, then for all languages $L \in B^*\mathbf{C}$:
$L\varphi^{-1} \in A^*\mathbf{C}$

- *An intersection of two language varieties also is a language variety.*
- *We say that a class of languages **C** generates a variety **V**, if **V** is the smallest variety, which contains **C**.*

Operations on languages: quotient

L – a language in an alphabet A, $a \in A$

$$a^{-1}L = \{v \in A^* \mid av \in L\}$$

$$La^{-1} = \{v \in A^* \mid va \in L\}$$

Operations on languages: morphisms

L_1 – a language in alphabet A, L_2 – a language in alphabet B

Morphism:

A function φ: $A^* \to B^*$, such that for all x, y $\in$ A^*

$$(xy)\varphi = (x\varphi)(y\varphi)$$

Therefore,

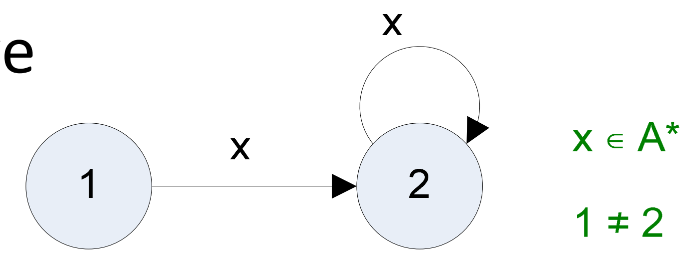
$$L_1\varphi = \{v \in B^* \mid \exists w \in L_1 : w\varphi = v\}$$

Inverse morphism:

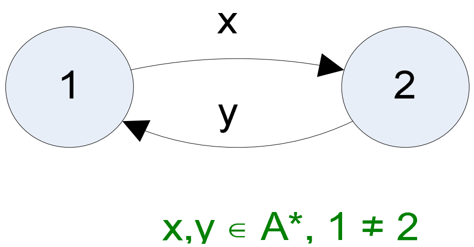
$$L_2\varphi^{-1} = \{w \in A^* \mid w\varphi \in L_2\}$$

Language varieties: examples

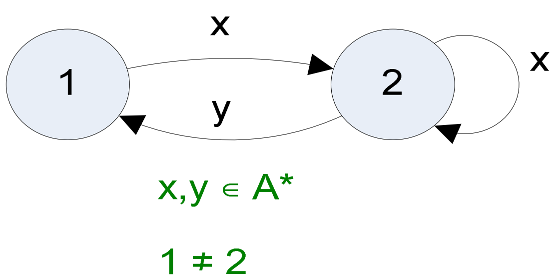
- Variety of groups **G**:
  min. det. automaton doesn't have the following construction:
  Deterministic Reversible Automata,
  Measure-Once Quantum Finite Automata



- Variety **R** (R-trivial languages):
  min. det. automaton doesn't have the following construction:

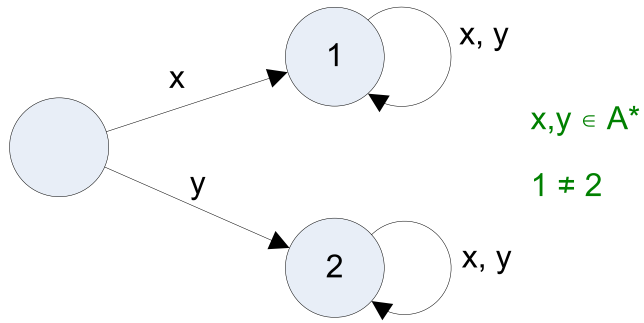


- Variety **R*G**:
  min. det. automaton doesn't have the following construction:

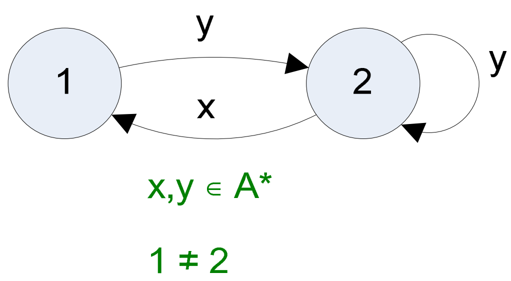


Language varieties: examples

- Variety **L*G**:
  min. det. automaton doesn't
  have the following construction:



- Variety **R*G**:
  min. det. automaton doesn't have
  the following construction:



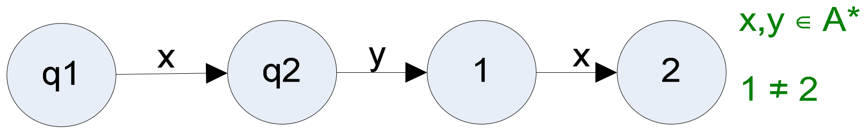
- Variety **BG = R*G ∩L*G**

  Classical Probabilistic Reversible Automata,

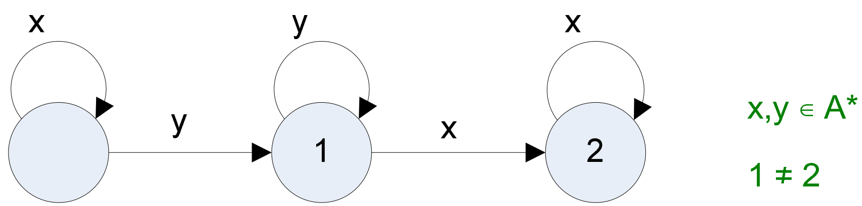
  Latvian Quantum Finite Automata

Language varieties: examples

- Variety $\mathbf{R_1}$
  (R-trivial idempotent languages):
  min. det. automaton doesn't have this construction.

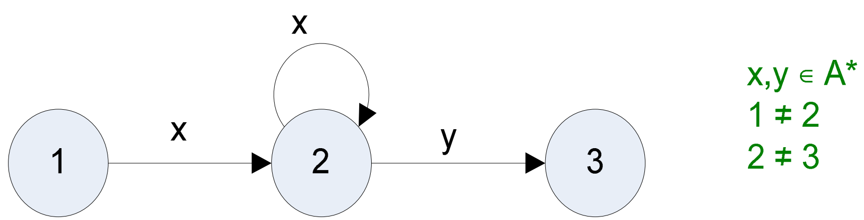


- Variety $\mathbf{R_1 * G}$:
  min. det. automaton doesn't have this construction.

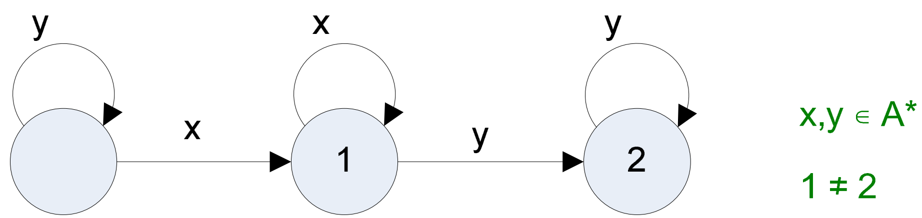


Decide-and-halt automata: RFA

- An RFA recognizes L iff the respective min. det. automaton doesn't have the following construction: [Ambainis, Freivalds 98]:

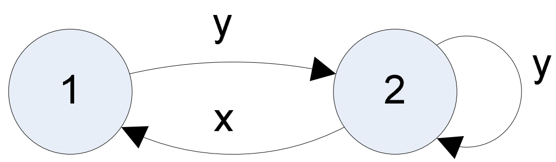


- The Boolean closure of RFA languages forms the language variety $\mathbf{R_1{*}G}$ (RFA *generates* $\mathbf{R_1{*}G}$).



Decide-and-halt automata: MM-QFA, DH-PRA, EQFA

- Languages don’t have the following forbidden construction (the forbidden construction of the first type):



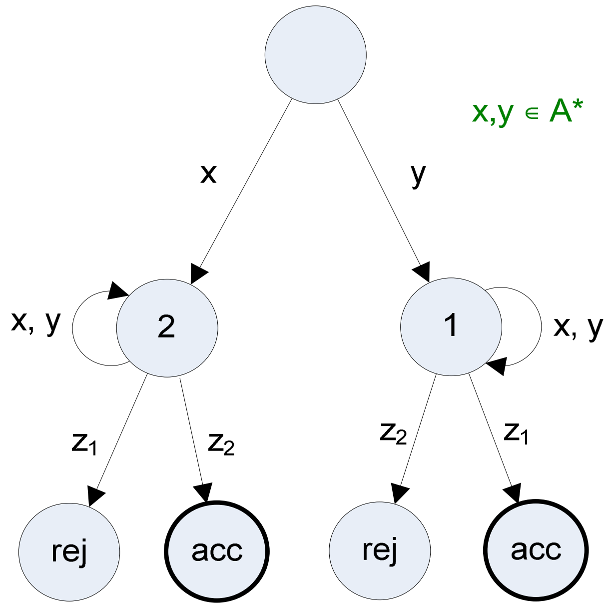
x,y ∈ A*, 1 ≠ 2

Hence they are contained in **R*G**.

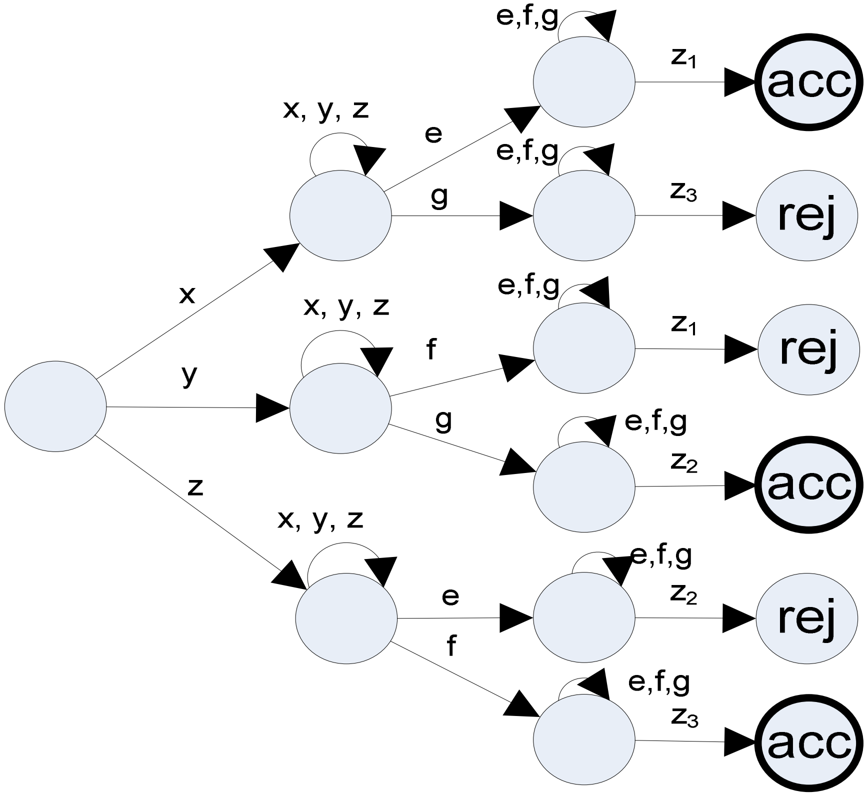
Decide-and-halt automata: MM-QFA, DH-PRA, EQFA

- Don't have a whole string of different forbidden constructions (thereafter – forbidden constructions of the second type), of whom the simplest one is the following:
  [Ambainis et al., Golovkins et. al., Mercer]

In this case it's not essential whether the deterministic automaton having a forbidden construction and recognizing a language is minimal or not.



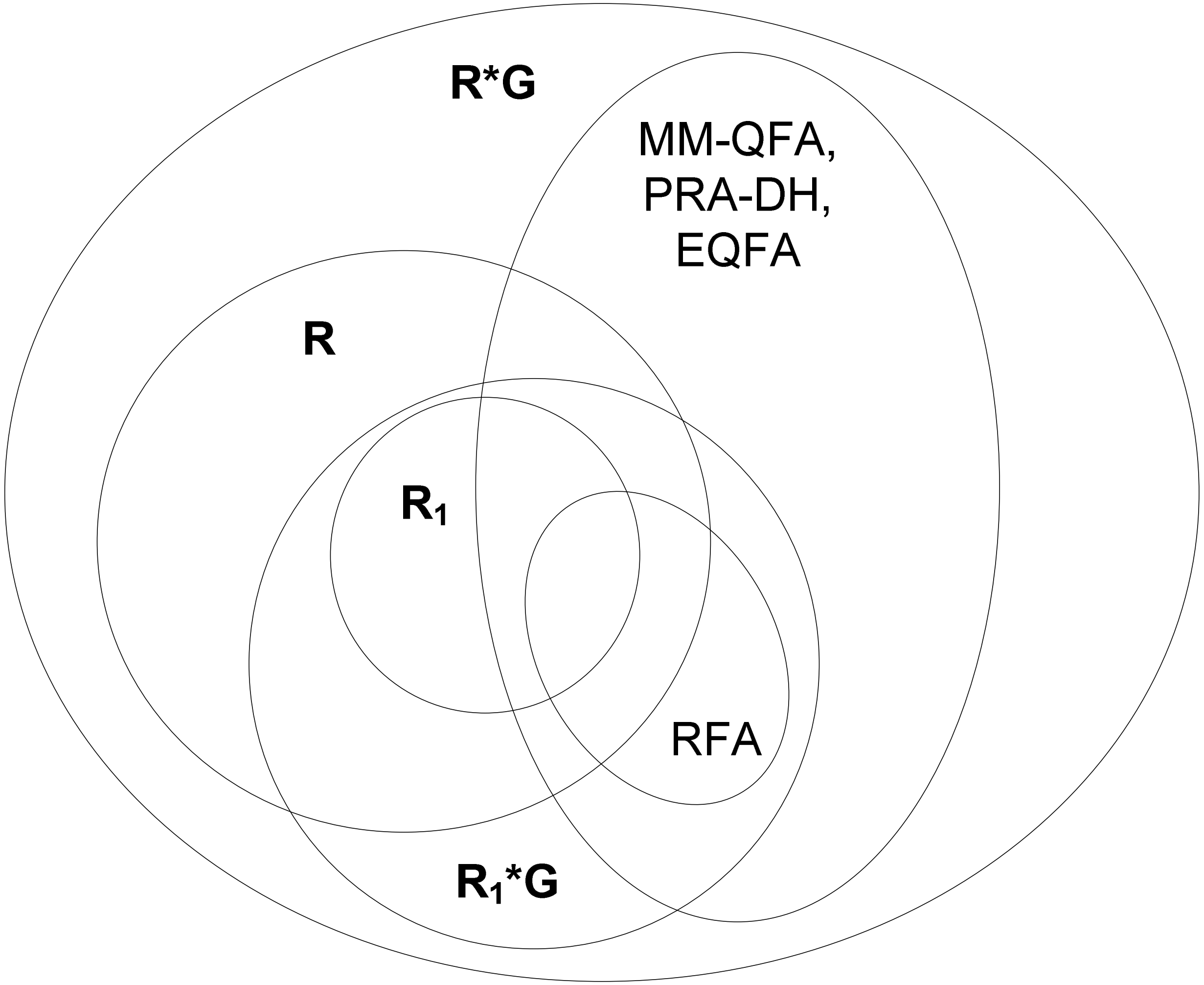
Decide-and-halt automata: forbidden constructions



Decide-and-halt automata: MM-QFA, DH-PRA, EQFA

Hypothesis. MM-QFA = DH-PRA = EQFA.

Decide-and-halt automata: MM-QFA, DH-PRA, EQFA



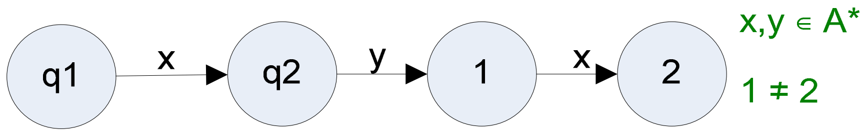
Decide-and-halt automata: MM-QFA, DH-PRA, EQFA

Research guidelines:

- Identify all the $\mathbf{R_1}$ languages that may be recognized by decide-and-halt automata.
- Identify all the R-trivial languages and $\mathbf{R_1{*}G}$ languages, that may be recognized by decide-and-halt automata.
- Identify all the **R*G** languages that may be recognized by decide-and-halt automata.

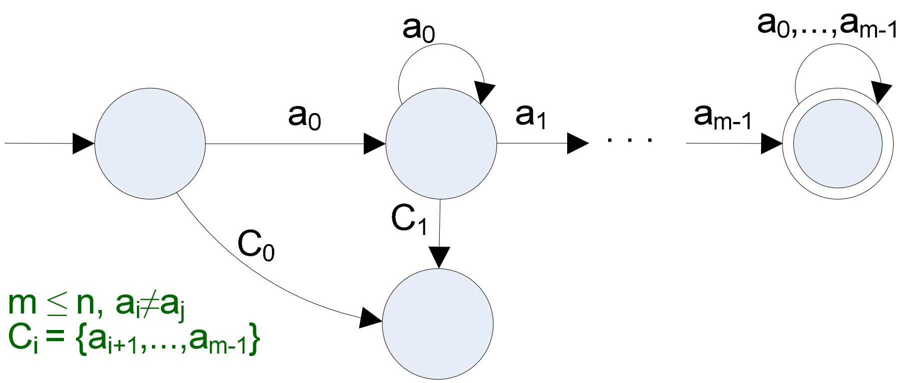
R-trivial idempotent languages (R_1 languages)

- Languages, that doesn't contain the following forbidden construction:



- Any R-trivial idempotent language in an alphabet of size n is a disjoint union of the following languages:

$a_0a_0{}^*a_1(a_0,a_1)^*\ldots a_{m-1}(a_0,a_1,\ldots, a_{m-1})^*$, where $m \le n$ and $i\neq j \rightarrow a_i\neq a_j$



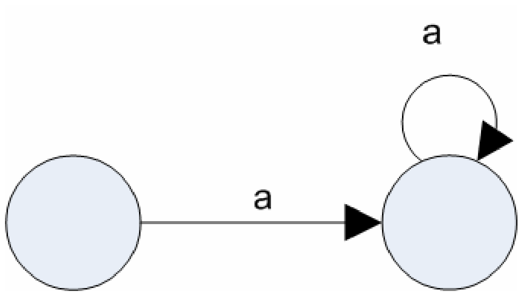
R-trivial idempotent languages

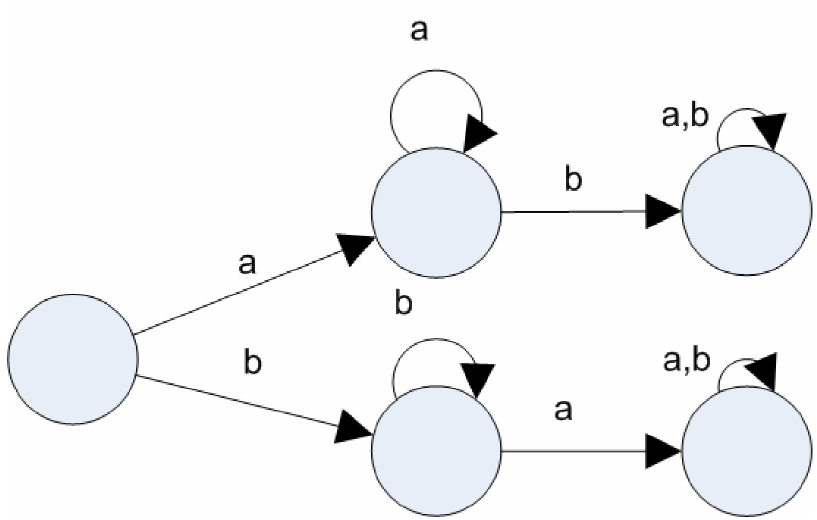
- Any R-trivial idempotent language in alphabet A is a Boolean closure of the following languages:

  $B^*a_iA^*$, where $B \subseteq A$ and $a_i \in A$.

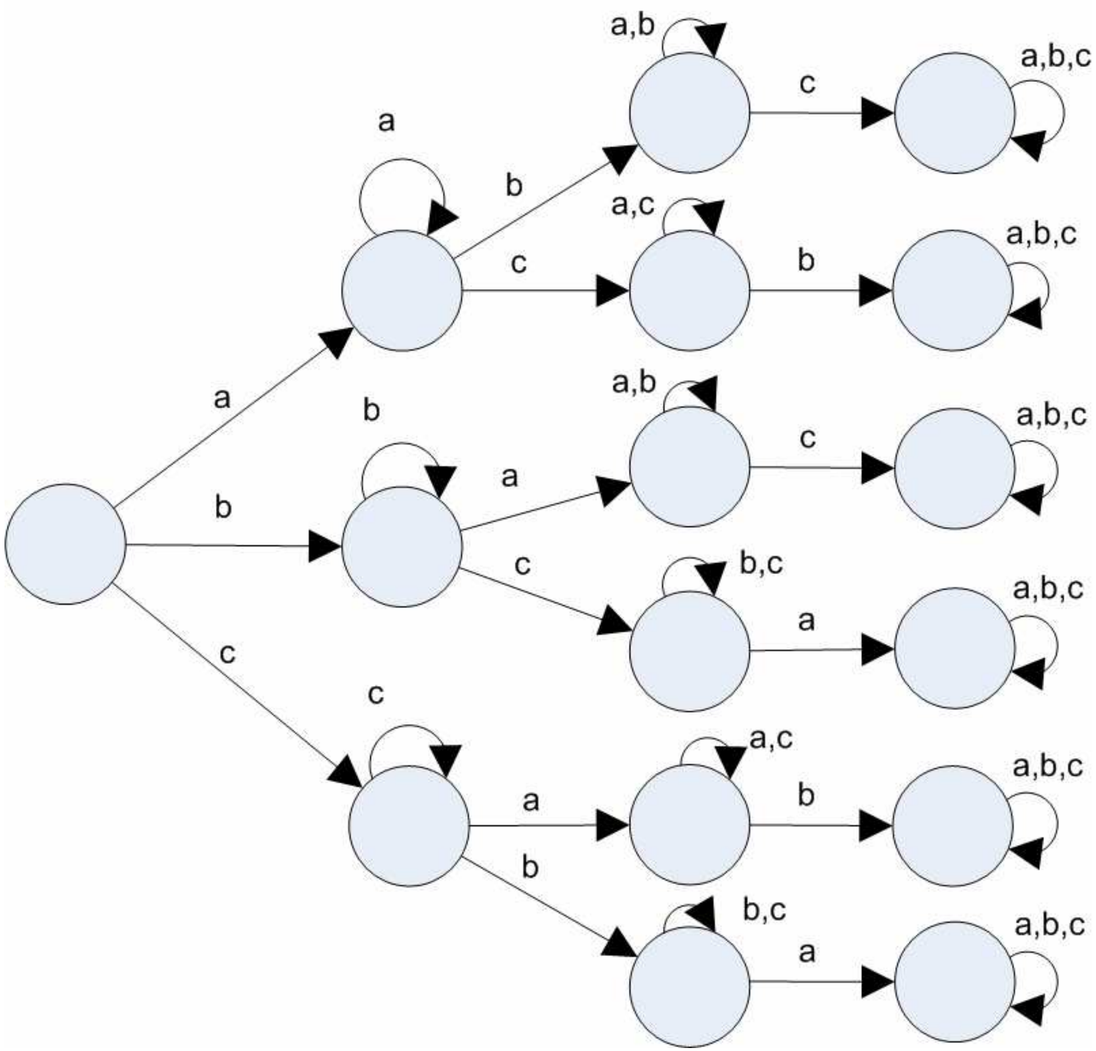
R-trivial idempotent languages

- Exists a deterministic finite automaton that can recognize any $\mathbf{R_1}$ language in a given alphabet.





R-trivial idempotent languages



R-trivial idempotent languages

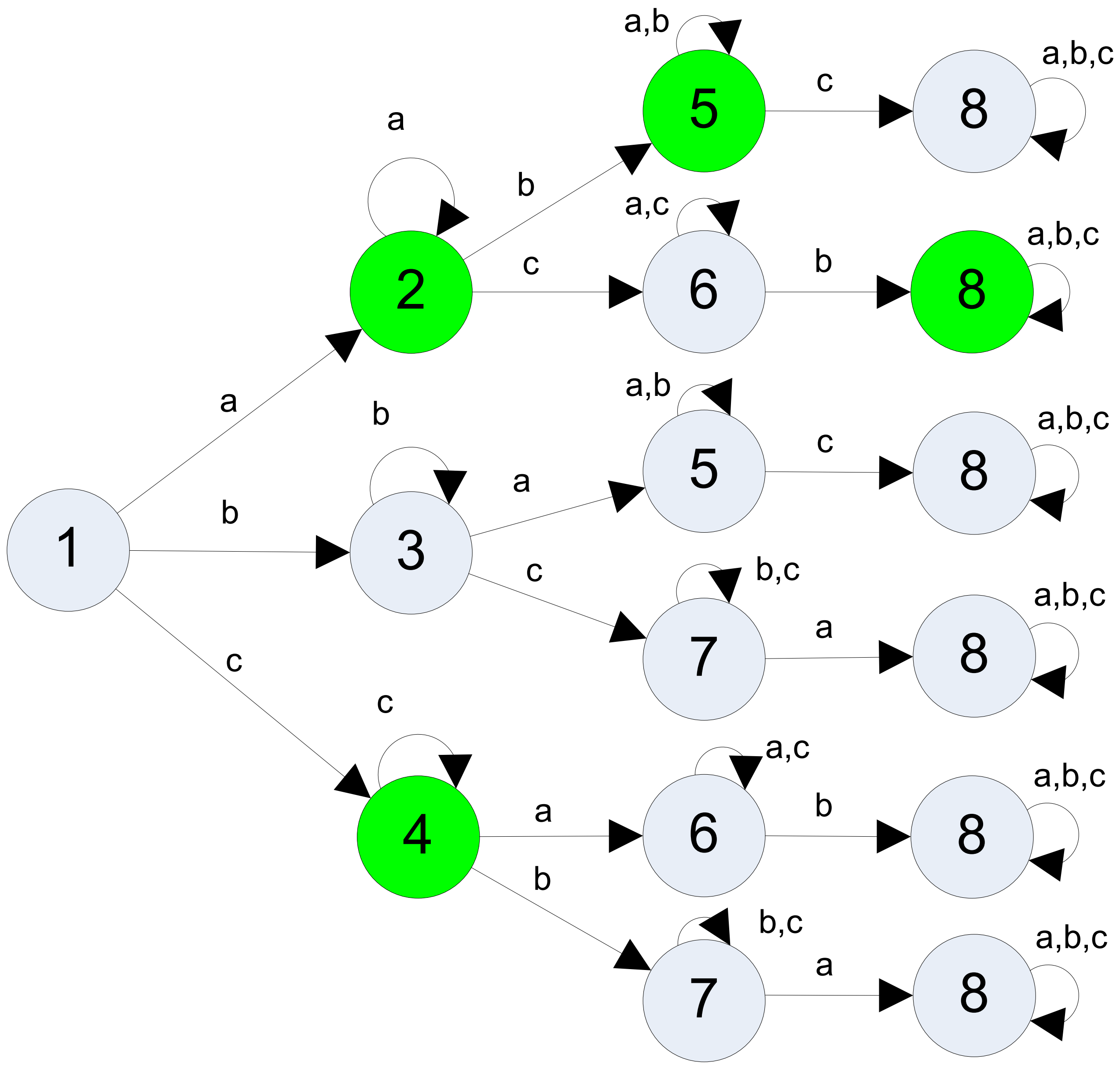
- For any R_1 language L, one may construct a linear system of inequalities with the following properties:

a) The system has a solution if and only if the language L is recognizable by PRA-DH.

b) The same system has a solution if and only if the language L is recognizable by QFA.

c) If the system has a solution, one may use the solution to construct a PRA-DH and a QFA that recognize the respective language.

Demo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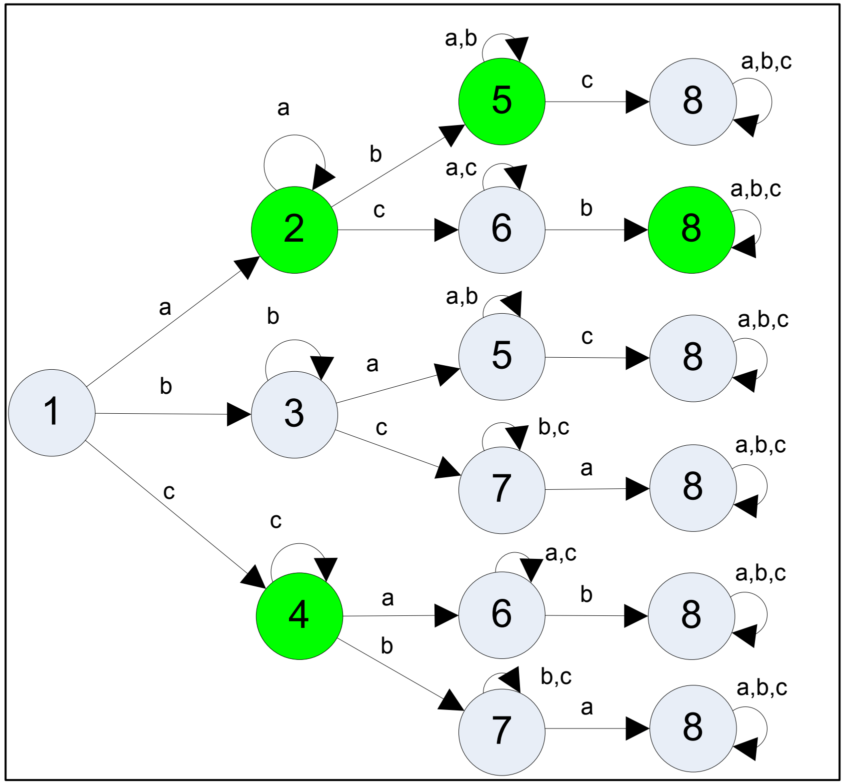
Demo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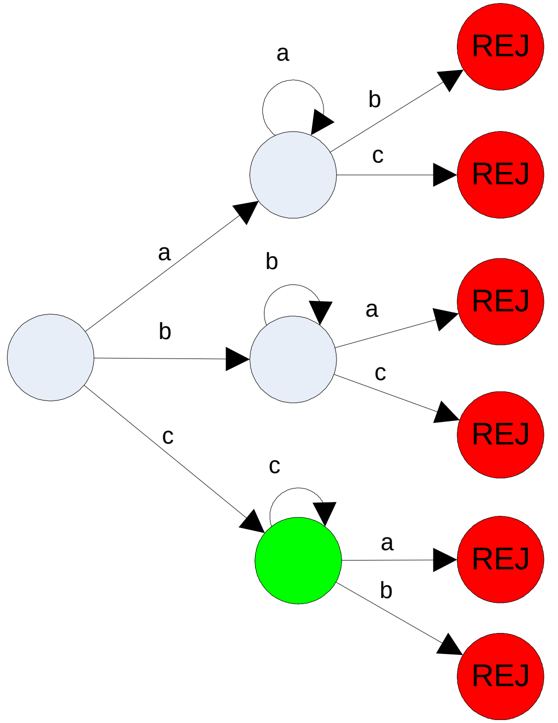
1/3


REJ
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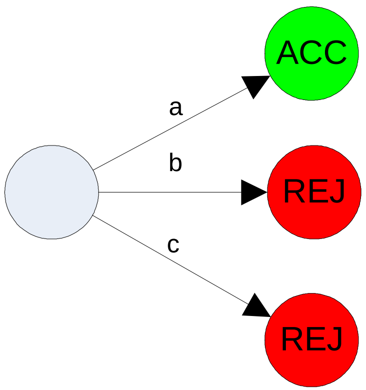
1/3


ACC
REJ
a
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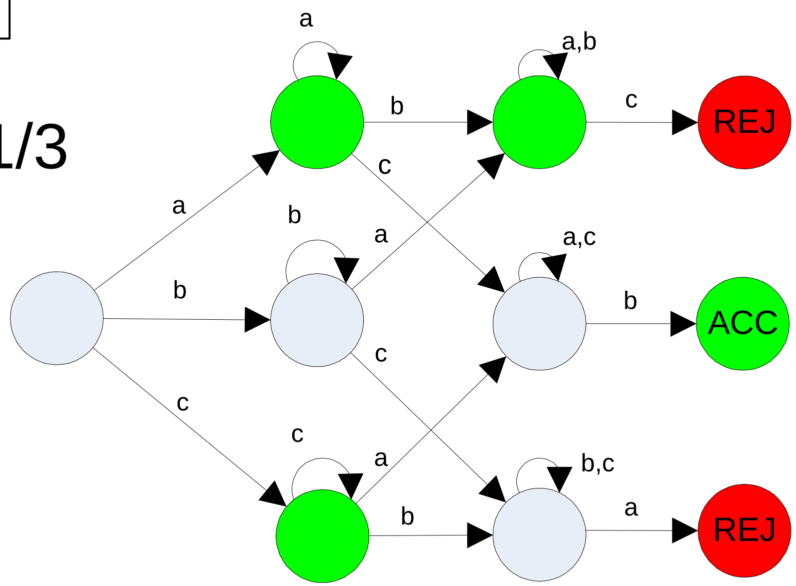
1/3


REJ
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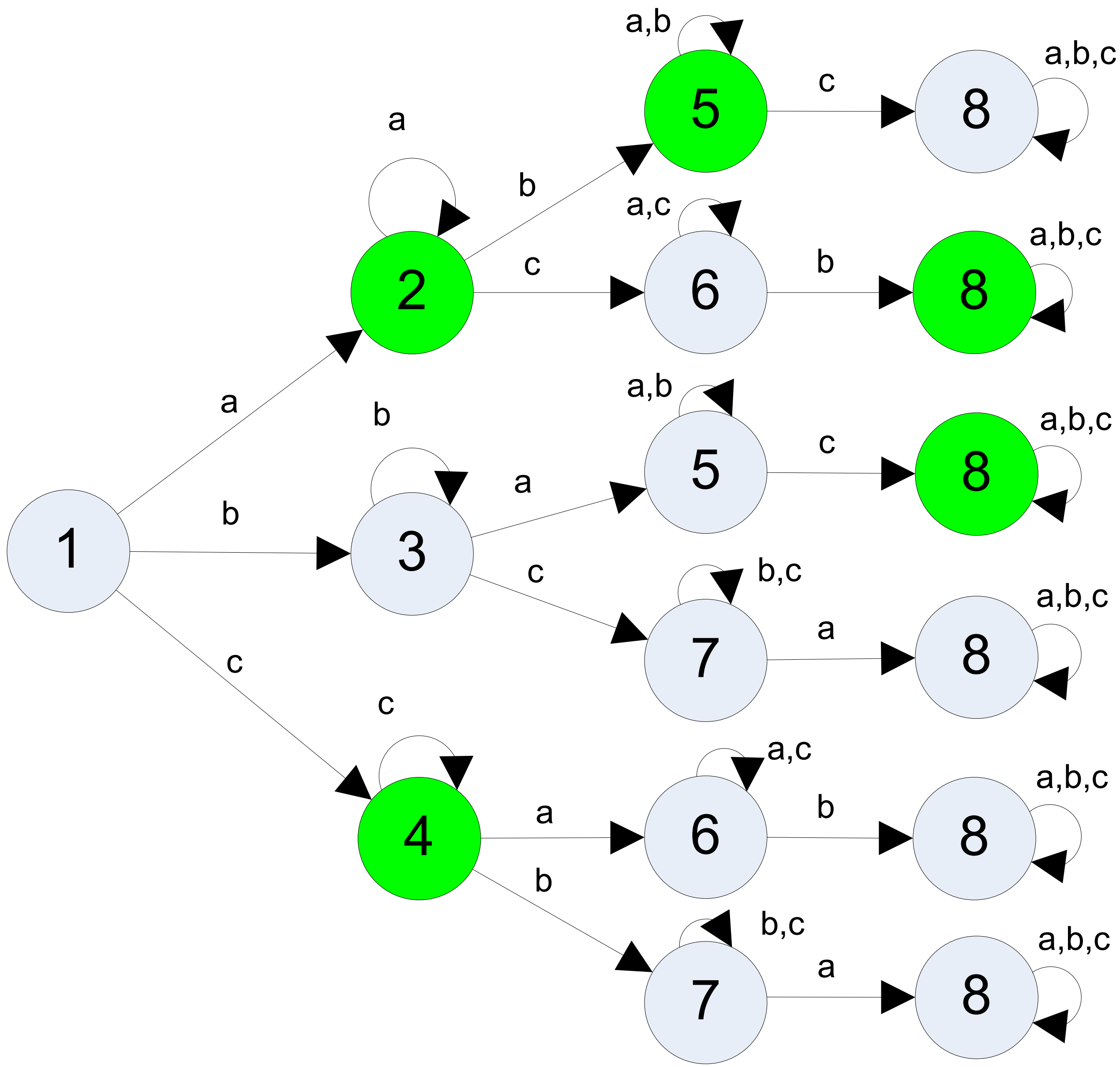
Demo



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R-trivial idempotent languages

- Theorem 1. For any $\mathbf{R_1}$ language L, it is decidable whether L can be recognized by PRA-DH or by QFA.
- Theorem 2. PRA-DH and QFA recognize the same set of R-trivial idempotent languages.

R-trivial idempotent languages:

The relation between forbidden constructions and system of inequalities

- If an R_1 language has a forbidden construction of Ambainis et.al., then the related system of linear inequalities is inconsistent.

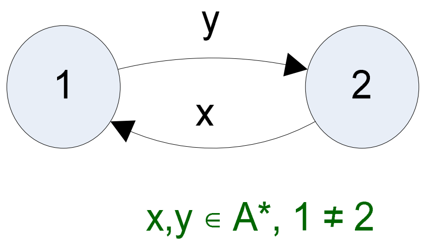
Decide-and-halt automata: MM-QFA, DH-PRA, EQFA

Research guidelines:

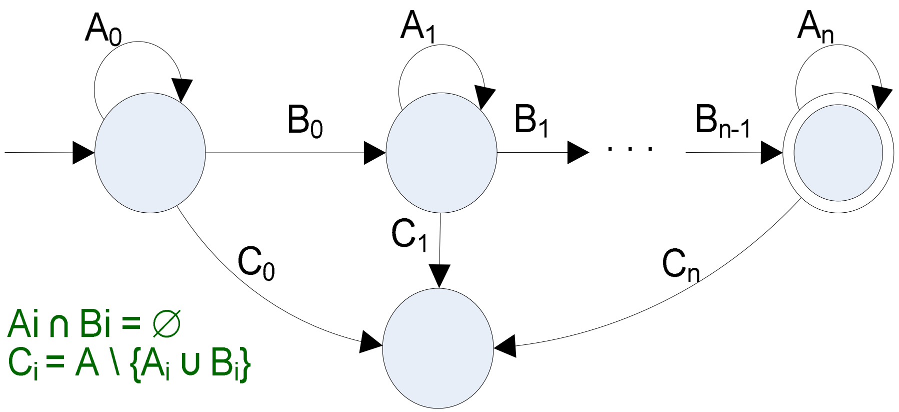
- Identify all the $\mathbf{R_1}$ languages that may be recognized by decide-and-halt automata.
- Identify all the R-trivial languages and R_1*G languages, that may be recognized by decide-and-halt automata.
- Identify all the **R*G** languages that may be recognized by decide-and-halt automata.

R-trivial languages

- Languages that don't have the forbidden construction:



- Any R-trivial language is a disjoint union of the following languages:



Decide-and-halt automata: MM-QFA, DH-PRA, EQFA

- Theorem 3. The Boolean closure of MM-QFA languages contains any R-trivial language. Similarly, DH-PRA un EQFA also generate any R-trivial language.

Results

- PRA-DH and MM-QFA recognize the same class of R-trivial idempotent languages.
- It is decidable whether MM-QFA recognize a given R_1 language.
- For any recognizable R_1 language, it is possible to construct the corresponding PRA-DH and MM-QFA by solving a system of linear inequalities.
- MM-QFA, PRA-DH, EQFA generate any R-trivial language;

Thank you!