

# SIGNATURES IN THE PRESENCE OF KEY- DEPENDENT MESSAGES

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# KDM-CPA Security

- Motivation: encrypting a hard drive which has the secret encryption key stored
- Adversary queries an encryption of a function of the secret key
- The goal is indistinguishability between encryptions of the secret key and random messages of the same length

# Signature Schemes

- Signing a backup of a hard disk which has the signing key stored
- Combined public key schemes
- Digital Signature Scheme Algorithms:
  - ▣ Key Generation
  - ▣ Signing Algorithm
  - ▣ Verification Algorithm

# Key Generation

- $K$  is a probabilistic key generation algorithm
- $K$  returns a pair of keys  $(sk, pk)$
- We interpret  $sk$  as the state of the signer



# Signing Algorithm

- $S$  is a signing algorithm
- On input  $M$  and state returns a signature  $\sigma$  or an error symbol  $\perp$ .
- Moreover, the state value  $sk$  is updated.



# Verification Algorithm

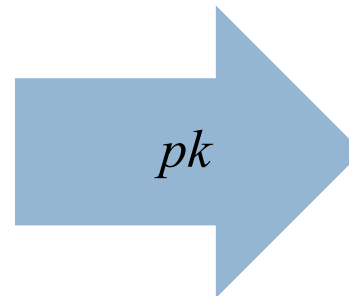
- $V$  is a deterministic verification algorithm
- On input a public key  $pk$ , a message  $M$ , and a candidate signature  $\sigma$  for  $M$  returns 1 or 0.



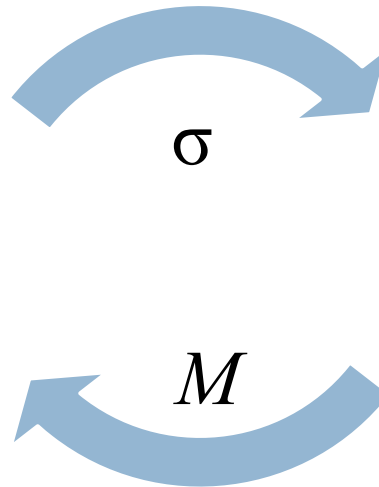
# EUFCMA Security

- Let  $(K, S, V)$  be a signature scheme and  $A$  an adversary

Run  $K$  and  
generate  
 $(sk, pk)$



# EUFCMA Security

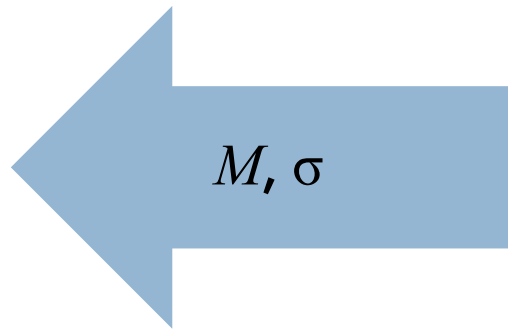


In the one-time EUFCMA setting,  $A$  gets only one query



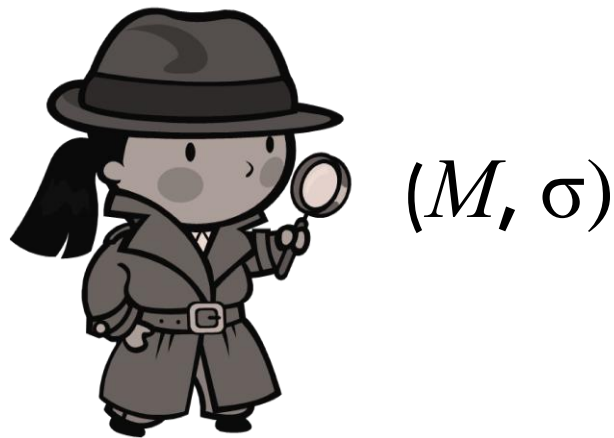


# EUFCMA Security



# EUFCMA Security

- $A$  wins if...
  - $V_{pk}(M, \sigma) = 1$
  - $M$  was not queried earlier



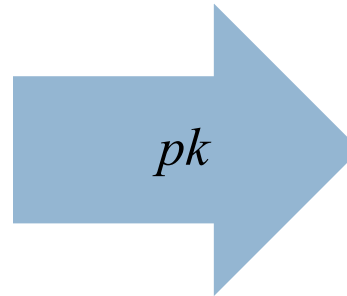
# KDS-CMA Security

Now the adversary can query a function  $g$  of the secret key

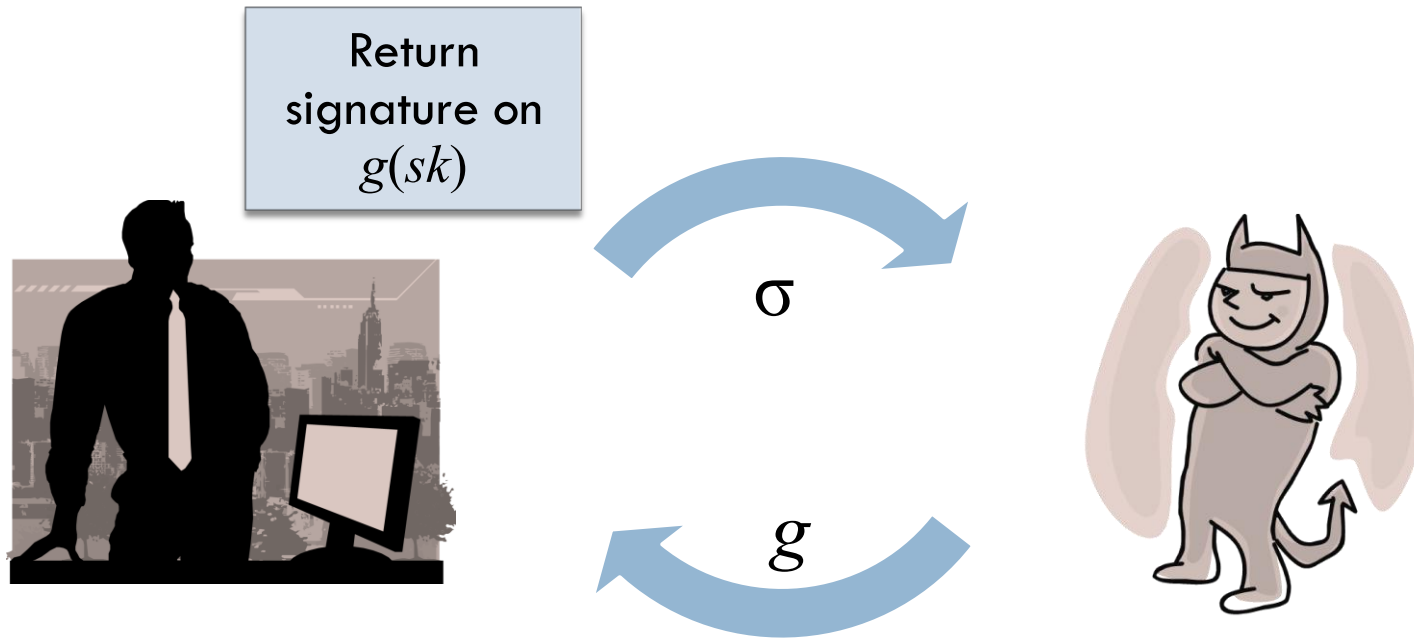
# KDS-CMA Security

- Let  $(K, S, V)$  be a signature scheme and  $A$  an adversary

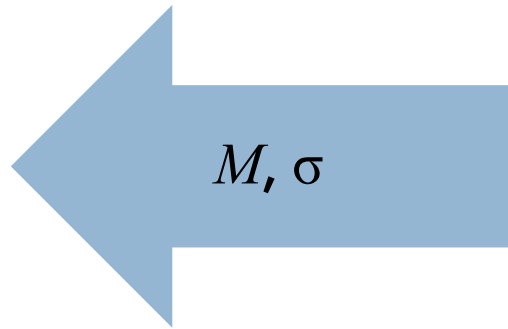
Run  $K$  and  
generate  $(sk, pk)$



# KDS-CMA Security



# KDS-CMA Security



# KDS-CMA Security

- $A$  wins if...
  - $V_{pk}(M, \sigma) = 1$
  - $M$  was not queried earlier (that is,  $M \neq g(sk)$  for any query)



$(M, \sigma)$

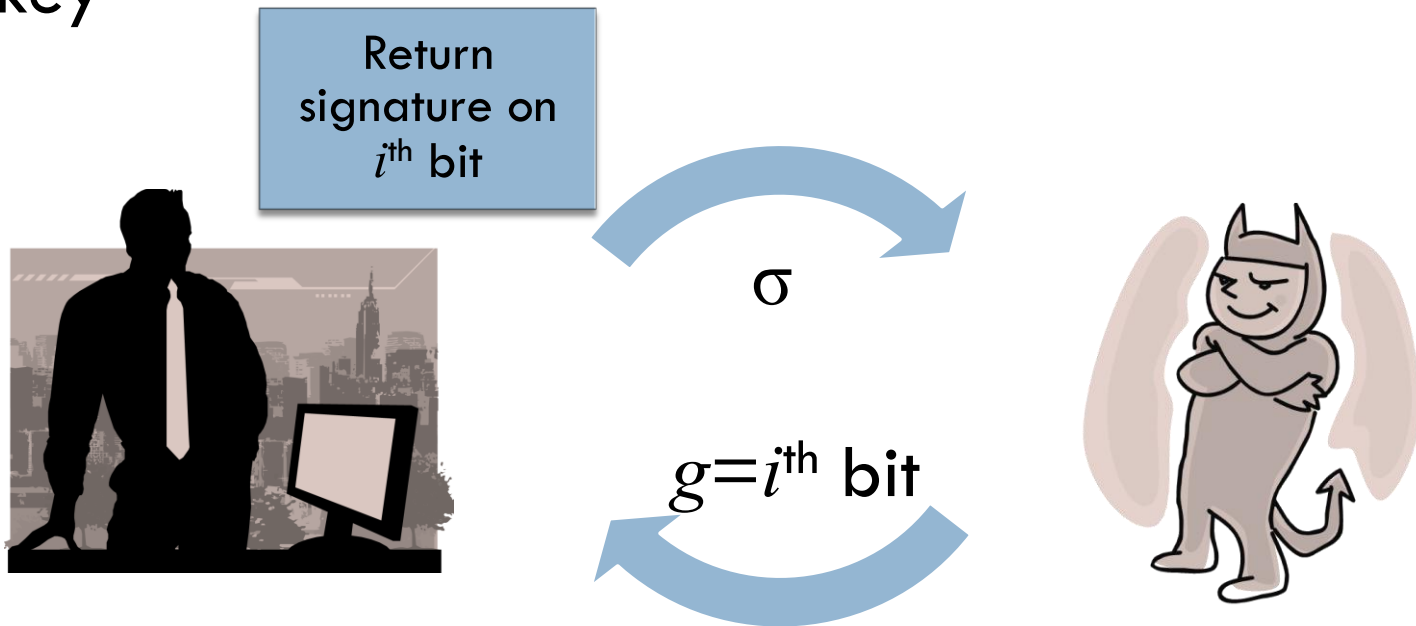
# Impossibility of Stateless Scheme

The secret key must be necessarily updated in this scenario

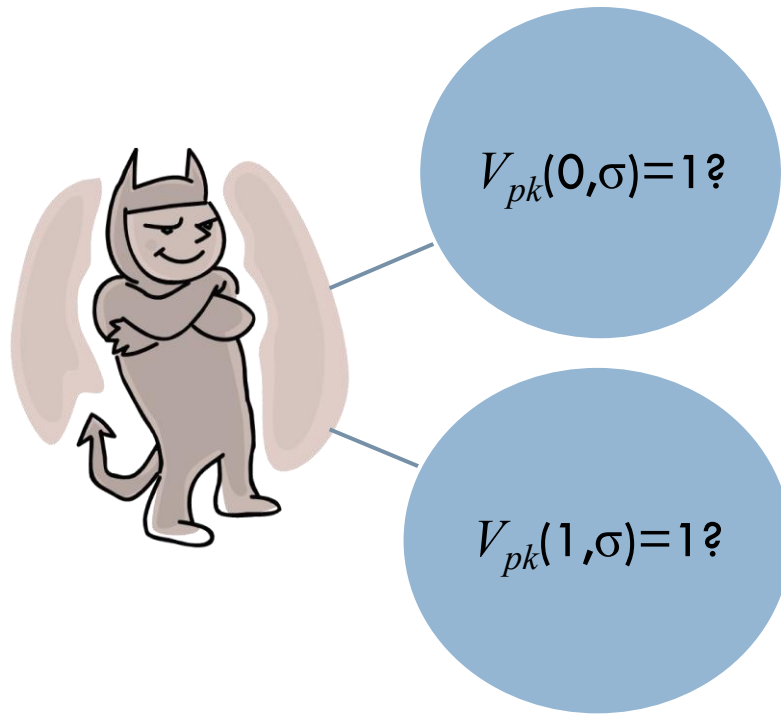


# Impossibility of Stateless Scheme

- Let  $sk = b_0 \dots b_{k-1}$  be the bit representation of a  $k$ -bit key



# Impossibility of Stateless Scheme



After  $k$  queries,  $A$  has the entire key, and this attack generalizes to a polynomial number of fixed keys.

# Forward Security

- Compromise of the secret key does not enable an adversary to forge signatures pertaining to the past
- In addition to  $(K, S, V)$ , there is an algorithm  $U$  which updates the secret key
- We adapt a forward secure scheme to build one secure in the KDS-CMA sense



# Long Signatures

## □ Key Generation

- Create pair  $(sk_0, pk_0)$
- $sk = (sk_0, [])$  for empty list  $[]$   
 $pk = pk_0$

## □ Key Update

- Create fresh key pair  $(sk_i, pk_i)$
- $\sigma_i = S_{sk_{i-1}}(i \parallel pk_i)$
- $Cert_i = (pk_i, \sigma_i)$
- New state is  $(sk_i, Certs)$

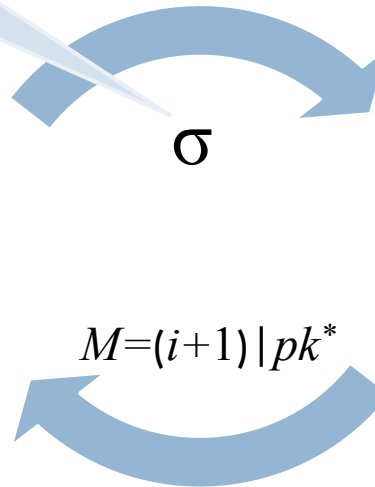
# Long Signatures

- Signing Algorithm
  - Let  $s = S_{sk_i}(M)$
  - Return  $(s, Certs)$
- Verification Algorithm
  - $V_{pk_i}(M, s) = 1$
  - $V_{pk_{j-1}}(pk_j, \sigma_j) = 1$   
for  $j = 1 \dots i$

# Long Signatures are Insecure

This  $\sigma$  is used to create  $Cert_{i+1}$  which  $A$  can use to forge by setting  $sk_{i+1} = sk^*$

Create  $(sk^*, pk^*)$  during period  $i$



# One-Time EUF-CMA to KDS-CMA Compiler

- Let  $(K, S, V)$  be a one-time EUF-CMA secure signature scheme
- Key Generation
  - ▣ Create pair  $(sk_0^c, pk_0^c)$
  - ▣ Will be used to create a certificate chain
  - ▣  $sk = (sk_0^c, \lambda, [])$  for empty string  $\lambda$  and empty list  $[]$   
 $pk = pk_0^c$

# One-Time EUF-CMA to KDS-CMA Compiler

## Signing Algorithm ( $i^{\text{th}}$ message)

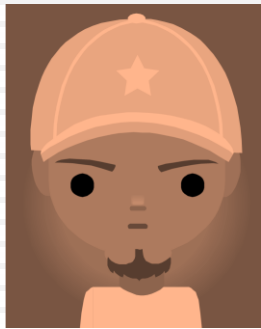
- Create two fresh key pairs  $(sk_i^c, pk_i^c)$  and  $(sk_i^m, pk_i^m)$
- $\sigma_i = S_{sk_{i-1}^c}(pk_i^c | pk_i^m)$
- $Cert_i = pk_i^c | pk_i^m | \sigma_i$
- $s = S_{sk_i^m}(r | H(M | r))$
- Signature is  $(r, s, Certs)$  and the state is updated

## Verification Algorithm

- $V_{pk_i^m}(r | H(M | r), s) = 1$
- $V_{pk_{j-1}^c}(pk_j^c | pk_j^m, \sigma_j) = 1$   
for  $j=1 \dots i$



# Sketch of the Proof



Let  $B$  be an  
EUF-CMA  
adversary

Let  $A$  be a KDS-  
CMA adversary

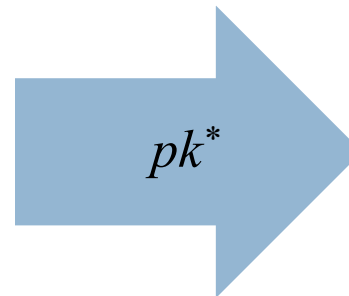


If  $A$  can forge,  $B$  can simulate and also forge

# Simulation by $B$

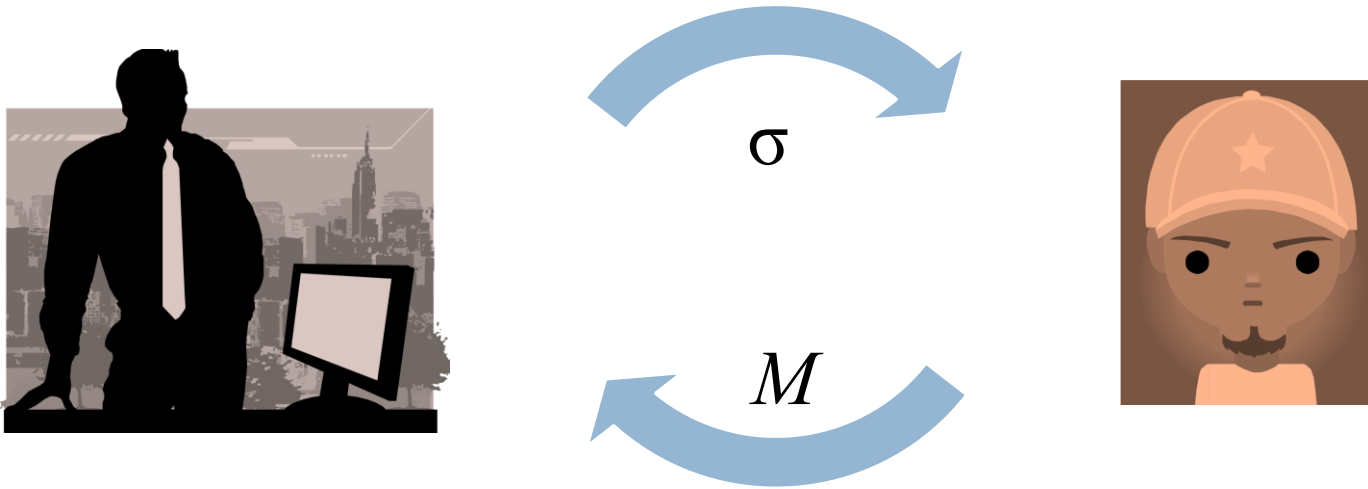
- Let  $(K, S, V)$  be a one-time signature scheme secure in the sense of EUF-CMA

Run  $K$  and  
generate  $(sk^*, pk^*)$



# Simulation by $B$

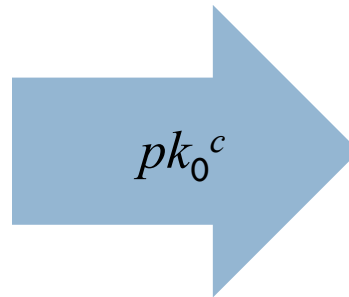
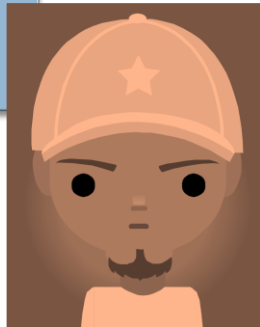
- $B$  gets one signature query under  $pk^*$



# Simulation by $B$

- Let  $(K', S', V')$  be the compiled signature scheme

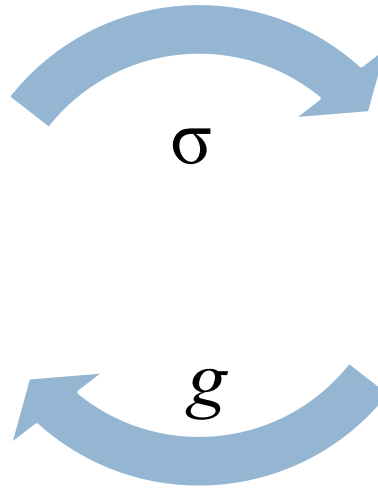
Run  $K'$  and  
generate  
 $(sk_0^c, pk_0^c)$



# Simulation by $B$

Create  $(sk_i^c, pk_i^c)$   
and  $(sk_i^m, pk_i^m)$

Return signature on  
 $g(sk)$



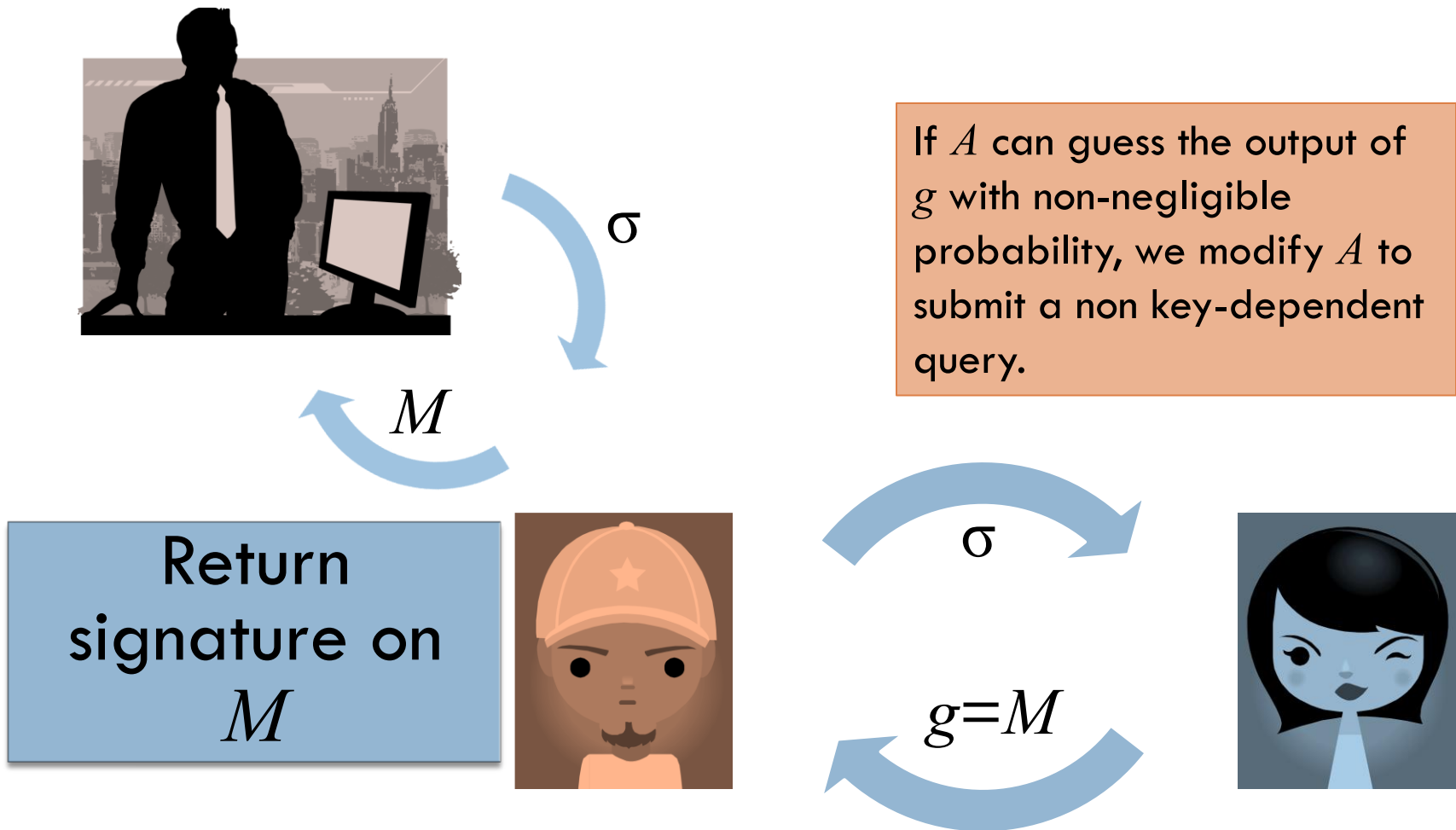
What about  $pk^*$ ?

# Simulation by $B$

- Two options:
  - Create  $(sk_i^c, pk_i^c)$  and set  $pk^* = pk_i^m$
  - Create  $(sk_i^m, pk_i^m)$  and set  $pk^* = pk_i^c$



# Simulation by $B$

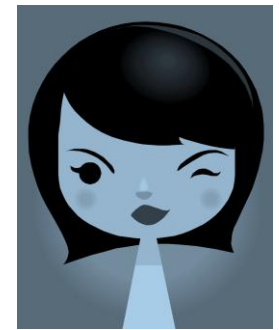
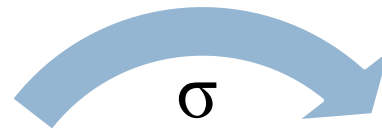
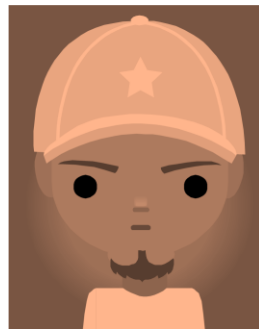


# Simulation by $B$



What if  $A$  cannot guess the output of  $g$  with non-negligible probability?

Create  $(sk', pk')$   
and set  $M = g(sk')$







Questions?