

Higher-Order Attribute Semantics of Flat Languages

Pavel Grigorenko
Joint work with Enn Tyugu

Institute of Cybernetics, Tallinn University of Technology

Rakari Theory Days

1 October 2010

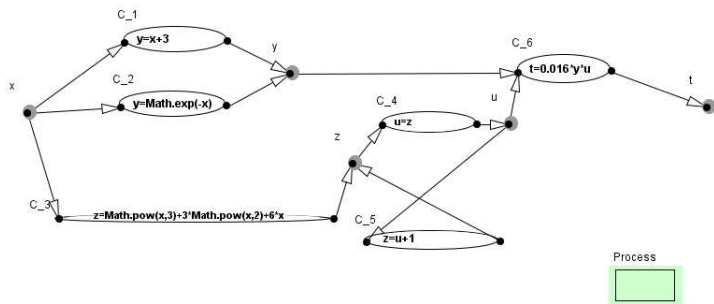


The image displays several windows from the CoCoViLa software:

- Top Left:** A large, complex network graph showing numerous interconnected nodes.
- Top Right:** A detailed flow diagram with nodes labeled VP, RVP, ZV, REL, RVT, PV, ME, RH, MH, VD, PIP, RVD, EH, LEH, WC.
- Middle Left:** A zoomed-in view of a network section with nodes like 'Propagator', 'Web-Server', 'DNS-Server', 'ACL-User', and 'Good guys'.
- Middle Right:** A 3D surface plot titled 'HydroPlot' showing a curved surface. The vertical axis is labeled 'step' and the horizontal axis is labeled 'cost'. A label 'coefficient of the load sensing system' is visible.
- Bottom Left:** A configuration panel for 'User Working' and 'Optimizer' with various checkboxes and a 'Section: C2IA1M2' label.
- Bottom Center:** A 'TableGrid' window showing a grid of cells and a keyboard layout.
- Bottom Right:** A 'GADAR' window showing a network diagram and a plot of 'Cost, Entropy' vs 'Success, Durability' with a highlighted point 'Response: [1] (0.05; 0.05)'.

Flat languages

Flat languages are declarative specification languages suitable for defining and composing objects into descriptions of concepts by binding their components by equalities (using *ports*).



(Screenshot from a paper by J. Sanko and J. Penjam)

Flat languages: types, classes

Compound types:

$$a : (t_1, \dots, t_n; a_1 : s_1, \dots, a_k : s_k),$$

Classes:

$$c : (s, (p_1, \dots, p_m), \mathfrak{S}),$$

- ▶ c – name
- ▶ s – type
- ▶ p_1, \dots, p_m – ports
- ▶ \mathfrak{S} – local semantics

Flat languages: syntax, semantics

Syntax:

$Statement ::= ObjectDeclaration | Binding$

$ObjectDeclaration ::= TypeName \ ObjectName$

$TypeName ::= Primitive | ClassName$

$Port ::= ObjectName | ObjectName.ComponentName$

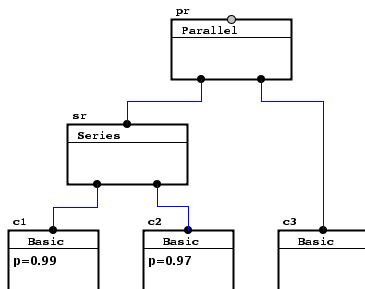
$Binding ::= Port = Port$

Semantics:

Local semantics \mathcal{G} + semantics of bindings (unfolding):

- a) Primitive types
- b) Compound types

Flat languages: computing reliability



Name of type	Supertype	Components	Computations
<i>double</i>			
<i>Basic</i>		<i>double p</i> <i>double q</i>	$p + q = 1$
<i>Parallel</i>	<i>Basic</i>	<i>Basic part1</i> <i>Basic part2</i>	$q = part1.q * part2.q$
<i>Series</i>	<i>Basic</i>	<i>Basic part1</i> <i>Basic part2</i>	$p = part1.p * part2.p$

Attribute models in a nutshell

- ▶ Attributes

$$a^\sigma \in A$$

- ▶ Attribute dependencies

$$r : x_1, \dots, x_m \rightarrow y_1, \dots, y_n\{f\}, r \in R$$

- ▶ Computational problems

$$G = U \rightarrow V, G \in P$$

- ▶ Higher-order attribute dependencies ($s \in P, x, y \in A$)

$$r^{ho} : s_1, \dots, s_k, x_1, \dots, x_m \rightarrow y_1, \dots, y_n\{f\}, r^{ho} \in R^{ho}$$

- ▶ Attribute model

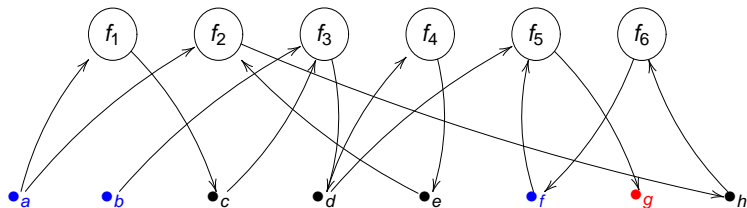
$$\langle A, R \cup R^{ho} \rangle$$



Attribute evaluation example

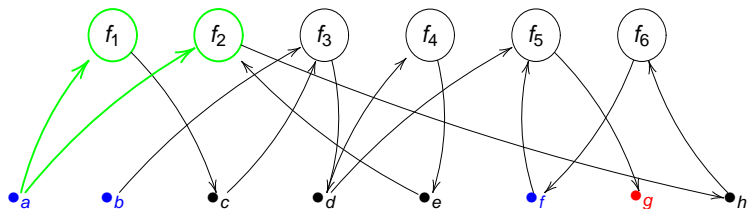
$$\langle A = \{a, b, c, d, e, f, g, h\},$$
$$R = \{a \rightarrow c\{f_1\};$$
$$a, e \rightarrow h\{f_2\};$$
$$b, c \rightarrow d\{f_3\};$$
$$d \rightarrow e\{f_4\};$$
$$d, f \rightarrow g\{f_5\};$$
$$h \rightarrow f\{f_6\}; \rangle$$

Value propagation



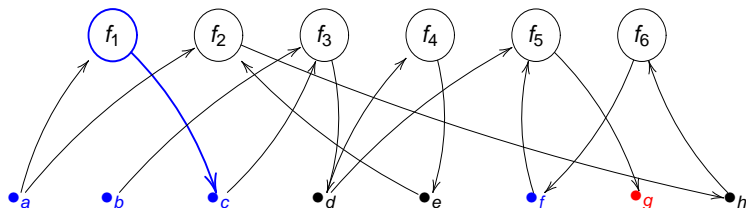
Computational problem $G = U : \{a, b, f\} \rightarrow V : \{g\}$

Value propagation



Computational problem $G = U : \{a, b, f\} \rightarrow V : \{g\}$

Value propagation

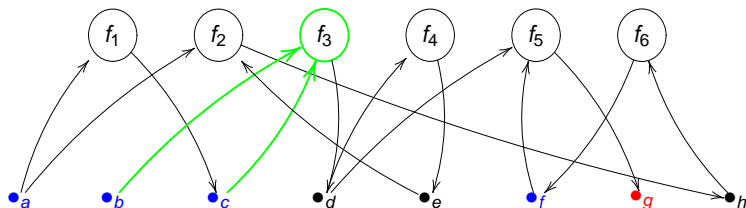


Computational problem $G = U : \{a, b, f\} \rightarrow V : \{g\}$

Algorithm:

$$c = f_1(a);$$

Value propagation

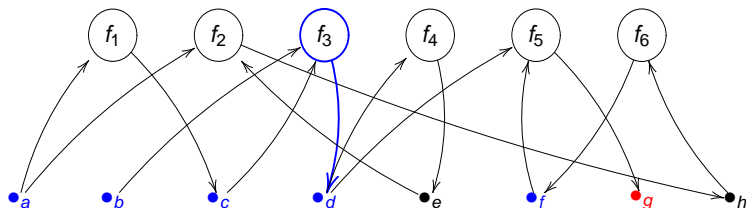


Computational problem $G = U : \{a, b, f\} \rightarrow V : \{g\}$

Algorithm:

$$c = f_1(a);$$

Value propagation



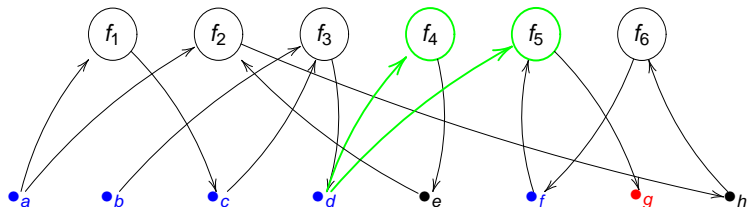
Computational problem $G = U : \{a, b, f\} \rightarrow V : \{g\}$

Algorithm:

$$c = f_1(a);$$

$$d = f_3(b, c);$$

Value propagation



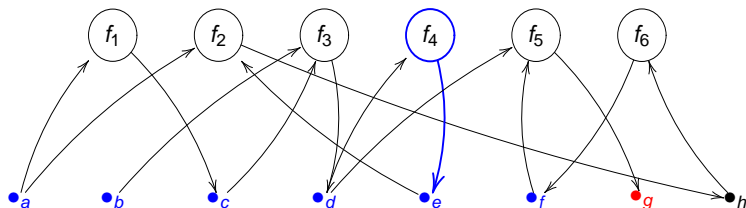
Computational problem $G = U : \{a, b, f\} \rightarrow V : \{g\}$

Algorithm:

$$c = f_1(a);$$

$$d = f_3(b, c);$$

Value propagation



Computational problem $G = U : \{a, b, f\} \rightarrow V : \{g\}$

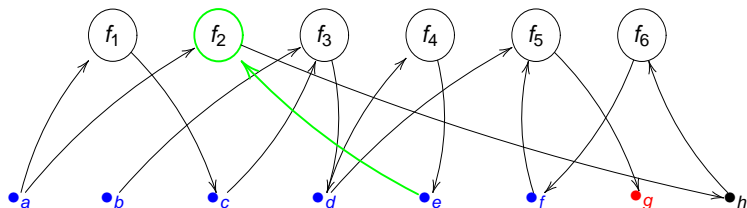
Algorithm:

$$c = f_1(a);$$

$$d = f_3(b, c);$$

$$e = f_4(d);$$

Value propagation



Computational problem $G = U : \{a, b, f\} \rightarrow V : \{g\}$

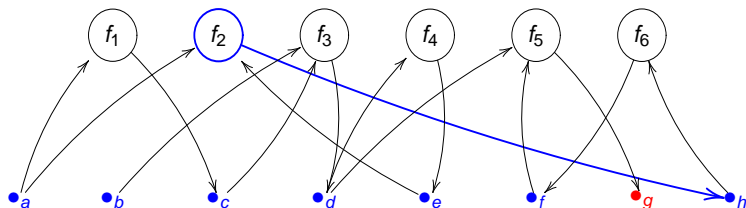
Algorithm:

$$c = f_1(a);$$

$$d = f_3(b, c);$$

$$e = f_4(d);$$

Value propagation



Computational problem $G = U : \{a, b, f\} \rightarrow V : \{g\}$

Algorithm:

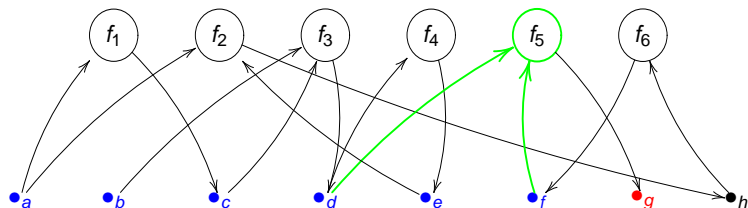
$$c = f_1(a);$$

$$d = f_3(b, c);$$

$$e = f_4(d);$$

$$h = f_2(a, e);$$

Value propagation



Computational problem $G = U : \{a, b, f\} \rightarrow V : \{g\}$

Algorithm:

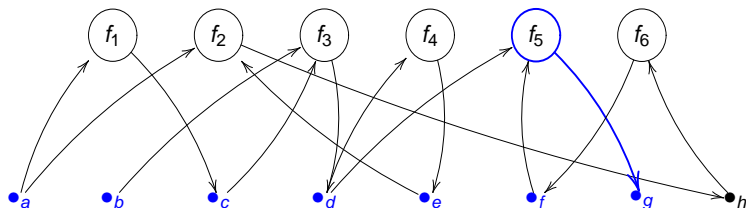
$$c = f_1(a);$$

$$d = f_3(b, c);$$

$$e = f_4(d);$$

$$h = f_2(a, e);$$

Value propagation



Computational problem $G = U : \{a, b, f\} \rightarrow V : \{g\}$

Algorithm:

$$c = f_1(a);$$

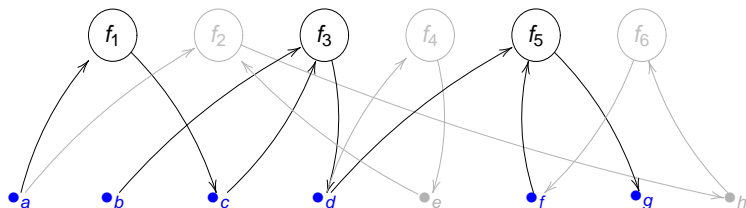
$$d = f_3(b, c);$$

$$e = f_4(d);$$

$$h = f_2(a, e);$$

$$g = f_5(d, f);$$

Optimization



Computational problem $G = U : \{a, b, f\} \rightarrow V : \{g\}$

Algorithm:

$$c = f_1(a);$$

$$d = f_3(b, c);$$

$$e = f_4(d);$$

$$h = f_2(a, e);$$

$$g = f_5(d, f);$$

Higher-order evaluation

Specification:

$$(a, b, f \rightarrow g), (a \rightarrow h), x \rightarrow y\{f_{ho}\}$$

Synthesized program:

$$y = f_{ho}(\lambda abf.f_5(f_3(b, f_1(a)), f), \lambda a.f_5(a), x)$$

Higher-order evaluation

Specification:

$$(a, b, f \rightarrow g), (a \rightarrow h), x \rightarrow y\{f_{ho}\}$$

Synthesized program:

$$y = f_{ho}(\lambda abf.f_5(f_3(b, f_1(a)), f), \lambda a.f_5(a), x)$$

Higher-order evaluation (2)

$$[y \rightarrow a] \rightarrow b$$
$$[a \rightarrow b] \rightarrow x$$
$$x, y \rightarrow a$$

Higher-order evaluation (2)

$S_1 : [y \rightarrow a]$

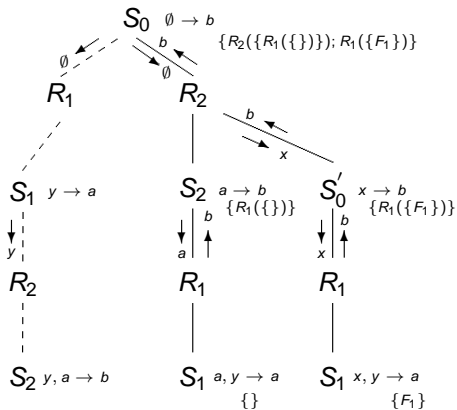
$S_2 : [a \rightarrow b]$

$R_1 : S_1 \rightarrow b$

$R_2 : S_2 \rightarrow x$

$F_1 : x, y \rightarrow a$

Solve: $\rightarrow b$



Theorem proving

$$((((A \rightarrow B) \rightarrow A) \rightarrow A) \rightarrow B) \rightarrow B$$

Theorem proving

$$((((A \rightarrow B) \rightarrow A) \rightarrow A) \rightarrow B) \vdash B$$

Theorem proving

$$((((A \rightarrow B) \rightarrow A) \rightarrow A) \rightarrow B) \vdash B$$

Equivalence replacement:

$$(X \leftrightarrow F) \rightarrow (G \leftrightarrow G_F[X])$$

$$\Gamma \vdash G[U \rightarrow V] \stackrel{ded}{=} X \rightarrow (U \rightarrow V), (U \rightarrow V) \rightarrow X, \Gamma \vdash G[X]$$

Theorem proving

$$((((A \rightarrow B) \rightarrow A) \rightarrow A) \rightarrow B) \vdash B$$

- ▶ Replace subformula $(A \rightarrow B)$ with X :

$$(A \rightarrow B) \rightarrow X,$$

$$X \& A \rightarrow B,$$

$$((X \rightarrow A) \rightarrow A) \rightarrow B \vdash B$$

Theorem proving

$$((((A \rightarrow B) \rightarrow A) \rightarrow A) \rightarrow B) \vdash B$$

- ▶ Replace subformula $(A \rightarrow B)$ with X :

$$\begin{aligned} &(A \rightarrow B) \rightarrow X, \\ &X \& A \rightarrow B, \\ &((X \rightarrow A) \rightarrow A) \rightarrow B \vdash B \end{aligned}$$

- ▶ Replace $(X \rightarrow A)$ with Y :

$$\begin{aligned} &(X \rightarrow A) \rightarrow Y, \\ &Y \& X \rightarrow A, \\ &(A \rightarrow B) \rightarrow X, \\ &X \& A \rightarrow B, \\ &(Y \rightarrow A) \rightarrow B \vdash B \end{aligned}$$

Theorem proving

$$((((A \rightarrow B) \rightarrow A) \rightarrow A) \rightarrow B) \vdash B$$

- ▶ Replace subformula $(A \rightarrow B)$ with X :

$$\begin{aligned} &(A \rightarrow B) \rightarrow X, \\ &X \& A \rightarrow B, \\ &(((X \rightarrow A) \rightarrow A) \rightarrow B) \vdash B \end{aligned}$$

- ▶ Replace $(X \rightarrow A)$ with Y :

$$\begin{aligned} &(X \rightarrow A) \rightarrow Y, \\ &Y \& X \rightarrow A, \\ &(A \rightarrow B) \rightarrow X, \\ &X \& A \rightarrow B, \\ &(Y \rightarrow A) \rightarrow B \vdash B \end{aligned}$$

Theorem proving

The sequent:

$$\begin{array}{c}
 (Y \& G \rightarrow A) \rightarrow B, \\
 (A \rightarrow (B \& G \& Z)) \rightarrow X, \\
 Y \& X \& U \rightarrow A, \\
 U, \\
 Z \vdash B
 \end{array}$$

Proof in GJ' (Mints):

$$\frac{\frac{\frac{A \vdash A}{A \vdash Y \& G \rightarrow A} \vdash \rightarrow \quad \frac{\frac{Z \vdash Z \quad G \vdash G}{Z, G \vdash G \& Z} \vdash \& \quad \frac{B \vdash B}{B, Z, G \vdash B \& G \& Z} \vdash \& \quad \frac{X \vdash X \quad \frac{Y \vdash Y \quad U \vdash U}{X, Y \vdash X \& Y} \vdash \&}{(Y \& G \rightarrow A) \rightarrow B, Z, G, A \vdash B \& G \& Z} \vdash \rightarrow \quad \frac{U, X, Y \vdash X \& Y \& U}{X \& Y \& U \rightarrow A, U, X, Y \vdash A} \vdash \& \quad \frac{A \vdash A}{A \vdash A} \vdash \rightarrow}{(Y \& G \rightarrow A) \rightarrow B, Z, G \vdash A \rightarrow (B \& G \& Z)} \vdash \rightarrow \quad \frac{A \vdash A}{A \vdash A} \vdash \rightarrow}{\frac{(Y \& G \rightarrow A) \rightarrow B, (A \rightarrow (B \& G \& Z)) \rightarrow X, X \& Y \& U \rightarrow A, U, Z, Y, G \vdash A}{(Y \& G \rightarrow A) \rightarrow B, (A \rightarrow (B \& G \& Z)) \rightarrow X, X \& Y \& U \rightarrow A, U, Z, Y \& G \vdash A} \& \vdash \quad \frac{(Y \& G \rightarrow A) \rightarrow B, (A \rightarrow (B \& G \& Z)) \rightarrow X, X \& Y \& U \rightarrow A, U, Z \vdash Y \& G \rightarrow A}{(Y \& G \rightarrow A) \rightarrow B, (A \rightarrow (B \& G \& Z)) \rightarrow X, X \& Y \& U \rightarrow A, U, Z \vdash Y \& G \rightarrow A} \vdash \rightarrow \quad \frac{B \vdash B}{B \vdash B} \vdash \rightarrow}{(Y \& G \rightarrow A) \rightarrow B, (A \rightarrow (B \& G \& Z)) \rightarrow X, X \& Y \& U \rightarrow A, U, Z \vdash B} \vdash \rightarrow}$$



Theorem proving

$$\begin{aligned} & (Y_1 \& G_1 \rightarrow A_1) \rightarrow B_1, \\ & (A_1 \rightarrow (B_1 \& G_1 \& Z_1)) \rightarrow X_1, \\ & Y_1 \& X_1 \& U_1 \rightarrow A_1, \\ & Z_1 \leftrightarrow B_2, \\ & Y_1 \leftrightarrow U_2, \\ & \dots \\ & (Y_n \& G_n \rightarrow A_n) \rightarrow B_n, \\ & (A_n \rightarrow (B_n \& G_n \& Z_n)) \rightarrow X_n, \\ & Y_n \& X_n \& U_n \rightarrow A_n, \\ & Z_{n-1} \leftrightarrow B_n, \\ & Y_{n-1} \leftrightarrow U_n, \\ & U_1, \\ & Z_n \vdash B_1, \end{aligned}$$

where $n \geq 2$

Benchmarking

Tool	<i>n</i> -level sequent							
	1	2	3	4	5	6	7	10
CoCoViLa (IDFS)	<0.01	<0.01	0.02	0.55	18.82	793.6	–	–
CoCoViLa (DFS)	<0.01	<0.01	<0.01	0.05	1.18	36.24	1579.6	–
STRIP (check)	<0.01	<0.01	<0.01	0.34	28.75	3781.3	–	–
STRIP (prove)	<0.01	34.87	–	–	–	–	–	–
iLeanCoP	0.01	–	–	–	–	–	–	–
iLeanSeP	0.02	–	–	–	–	–	–	–
iLeanTAP	–	–	–	–	–	–	–	–
LJT	<0.01	<0.01	0.01	0.05	1.04	35.15	1572.28	–
Gandalf	0.01	0.02	0.08	0.19	0.36	0.53	0.88	7.550
PITP	<0.01	0.01	0.01	0.05	0.68	15.73	343.5	–

Conclusions

- ▶ Introduced a concept of *flat languages*
- ▶ Represented *semantics* by means of attribute models
- ▶ Implemented the approach in CoCoViLa¹

¹ www.cs.ioc.ee/cocovila