Solutions for CHSH (Clauser - Horne - Shimony - Holt) game

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What will you hear

- ✓ Intro about CHSH
- ✓ Memory loophole
- ✓ Combinatorial solutions

The Game

Definition

Input: x and y
Answer: xr and yr
Both players win if $x \wedge y \equiv xr \oplus yr$, else they loose.

	x1(0)	x2(1)
y1(0)	$(x1 \oplus y1)?=0$	$(x2 \oplus y1)?=0$
y2(1)	$(x1 \oplus y2)?=0$	$(x2 \oplus y2)?=1$

Classical solution

	0(0)	0(1)
0(0)	(0 ⊕ 0)=0	(0 ⊕ 0)=0
0(1)	(0 ⊕ 0)=0	(0 ⊕ 0)!=1

Win probability is 0.75

Quantum solution

Starting quantum system: $|w\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}$

Measurment base rotation:

Players → Input ↓	First	Second
0	$-\frac{\pi}{16}$	$\frac{\pi}{16}$
1	$\frac{3\pi}{16}$	$-\frac{3\pi}{16}$

Win probability ~0.85

Memory Loophole

Parallel CHSH games

- Using quantum mechanics it is $(0.85)^n$, proved by R.Cleve, W. Slofstra, F. Unger, S. Upadhyay
- For classical solution it would normally be $(0.75)^{n,}$ but it is proven that solution is better
  - 2 parallel games is $10/16 > (3/4)^2$
  - 3 parallel games is 31/64
  - 4 parallel games is $(10/16)^2$ (experimentally proven)
  - n parallel games $\sim 0.809^n$ (our result $\sim 0.7937^n$, naive result 0.7905^n)

Classical solution 2 parallel games

Answer (Input)	00(00)	00(01)	00(10)	10(11)
00(00)	00	00	00	10
00(01)	00	01	00	11
00(10)	00	00	10	00
01(11)	01	00	11	00

Win probabillity 10/16

Why best?

(00)	00	00	00	00
(11)	00	01	10	11
Difference	00	01	10	11

(01)	00	01	00	01
(10)	00	00	10	10
Difference	00	01	10	11

Each part has max 5 correct answers

Classical solution 3 parallel games

Answer (Input)	000 (000)	001 (001)	010 (010)	001 (011)	100 (100)	001 (101)	110 (110)	110 (111)
001 (000)	001	000	011	000	101	000	111	111
000 (001)	000	000	010	000	100	000	110	111
100 (010)	100	101	100	111	000	101	000	000
000 (011)	000	000	000	010	100	000	100	101
010 (100)	010	011	000	011	010	111	000	000
000 (101)	000	000	010	000	000	100	010	011
000 (110)	000	001	000	011	000	101	000	000
000 (111)	000	000	000	010	000	100	000	001

Win probability 31/64

Proof for 3 parallel games (I)

Split in two isomorphic parts:

	(000)	**(001)**	**(010)**	**(011)**	**(100)**	**(101)**	**(110)**	**(111)**
(000)	(000)	(000)	(000)	(000)	(000)	(000)	(000)	(000)
(101)	(000)	(001)	(000)	(001)	(100)	(101)	(100)	(101)
(110)	(000)	(000)	(010)	(010)	(100)	(100)	(110)	(110)
(011)	(000)	(001)	(010)	(011)	(000)	(001)	(010)	(011)

	(000)	**(001)**	**(010)**	**(011)**	**(100)**	**(101)**	**(110)**	**(111)**
(100)	(000)	(000)	(000)	(000)	(100)	(100)	(100)	(100)
(001)	(000)	(001)	(000)	(001)	(000)	(001)	(000)	(001)
(010)	(000)	(000)	(010)	(010)	(000)	(000)	(010)	(010)
(111)	(000)	(001)	(010)	(011)	(100)	(101)	(110)	(111)

Proof for 3 parallel games (II)

- Adding some values to player answers changes table stucture

	(000)	**(001)**	**(010)**	**(011)**	**(100)**	**(101)**	**(110)**	**(111)**
000 (000)	(000)	(000)	(000)	(000)	(000)	(000)	(000)	(000)
001 (101)	(001)	(000)	(001)	(000)	(101)	(100)	(101)	(100)
000 (110)	(000)	(000)	(010)	(010)	(100)	(100)	(110)	(110)
001 (011)	(001)	(000)	(011)	(010)	(001)	(000)	(011)	(010)

Proof for 3 parallel games (III)

- Use claims:
  - Possible to change first column and any other
  - Possible to change first row and any other
  - Same value can be added to all row answers

Proof for 4 parallel games

- Split table in:
  - (0000, 0011, 0110, 0101, 1001, 1010, 1100, 1111) rows;
  - (0001, 0010, 0100, 0111, 1000, 1011, 1101, 1110) rows.
- In each table is possible to calculate max value as 50/128

N parallel games

solved by

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Algorithm

1. While a_x have been chosen for less than $2^n/4$ values $x \in \{0, 1\}^n$:

   a) Find an $x \in \{0, 1\}^n$ satisfying $|\{t | d(x,t) = n/3 \text{ and } a_t \text{ is not fixed }\}| \geq \frac{3\binom{n}{n-3}}{4}$

   b) For each $t \in \{0,1\}^n$ such that $d(x,t) = \frac{n}{3}$ and v_t is not fixed, set $v_t = x \wedge t$.

2. Choose the remaining a_x arbitrarily

Idea!

Claim:

If, for every i ∈ {1,...,N}, we have $x_i = y_i$ or $x_i = z_i$, then

$(z \wedge x) \oplus (z \wedge y) = (x \wedge y) \oplus x$

	(x)	(y)	(z)
x (x)	$(x \wedge x) \oplus x$	$(y \wedge x) \oplus x$	$(z \wedge x) \oplus x$
x^y (y)	$(x \wedge y) \oplus (x \wedge y)$	$(y \wedge y) \oplus (x \wedge y)$	$(z \wedge y) \oplus (x \wedge y)$
x^z (z)	$(x \wedge z) \oplus (x \wedge z)$	$(y \wedge z) \oplus (x \wedge z)$	$(z \wedge z) \oplus (x \wedge z)$
x^a (a)	$(x \wedge a) \oplus (x \wedge a)$	$(y \wedge a) \oplus (x \wedge a)$	$(z \wedge a) \oplus (x \wedge a)$
	= 0	$=(y \wedge x) \oplus x$	$=(z \wedge x) \oplus x$

Possible to choose $\binom{n}{n/3}$ *columns and* $\binom{2n/3}{n/3}$ *row values are identical in each column*

Result

1. Each cycle has $\frac{1}{2}\binom{2n/3}{n/3}\binom{n}{n/3}$
2. Cycles is $\dfrac{2^n}{4\binom{n}{n/3}}$
3. Result is $\dfrac{2^n}{4\binom{n}{n/3}}\dfrac{1}{2}\binom{2n/3}{n/3}\binom{n}{n/3} = \Omega\left(\dfrac{2^{5n/3}}{\sqrt{n}}\right)$

Disscusion

- ✓How to prove 4 game best result?
- ✓How to solve 5 games experimentally?
- ✓How to improve n parallel games?
- ✓Why in 4 games naïve lower bound is best?

Thanks!

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