A Framework for bounding nonlocality of state discrimination

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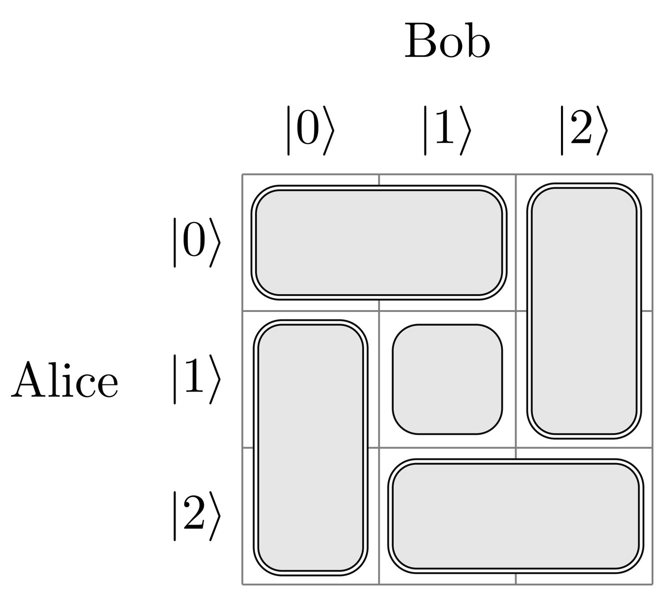
Motivation

- Understand LOCC and separable operations and difference between them.
- Develop new tools for working with LOCC protocols. In particular, for lower bounding the error probability.

State Discrimination Problem

Let $S = \{|\psi_1\rangle, \dots, |\psi_n\rangle\} \subset \mathbb{C}^{d_A} \otimes \mathbb{C}^{d_B}$ be a known set of quantum states. Suppose that $k \in \{1, \dots, n\}$ is selected uniformly at random and Alice and Bob are given the corresponding parts of state $|\psi_k\rangle \in S$. Their task is to determine the index k.

Quantum Nonlocality Without Entanglement

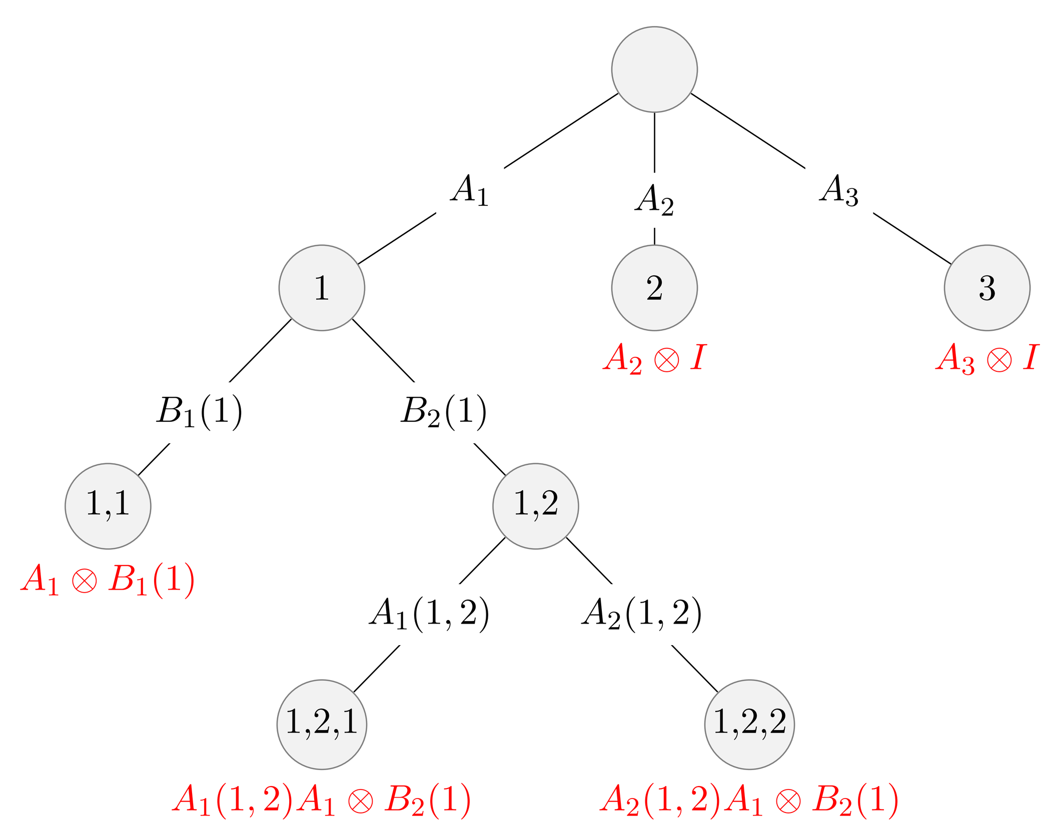


Theorem (see [1]). Any LOCC protocol for discriminating states

$$|1\rangle|1\rangle$$
$$|0\rangle|0 \pm 1\rangle \quad |1 \pm 2\rangle|0\rangle$$
$$|2\rangle|1 \pm 2\rangle \quad |0 \pm 1\rangle|2\rangle$$

has mutual information deficit at least 0.00000531 bits.

LOCC Protocol As a Tree



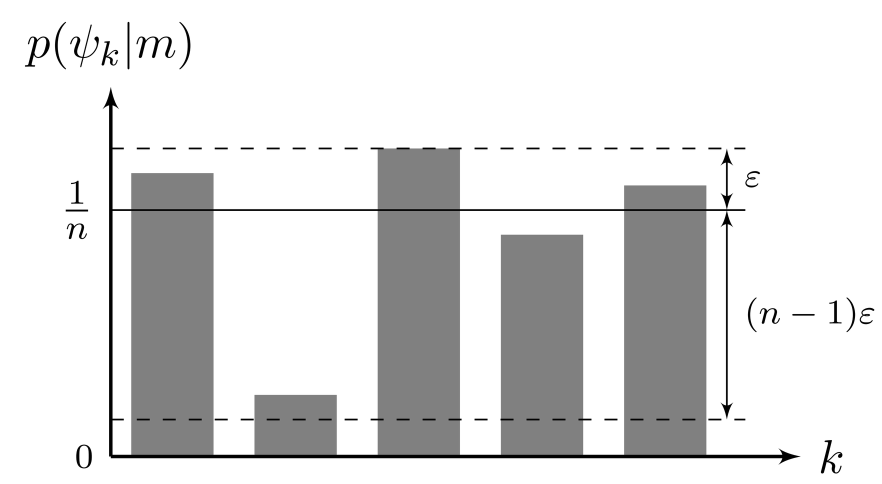
Nonlocality Constant

Definition. Let $G_{ij} = \langle\psi_i|(a \otimes b)|\psi_j\rangle$ for some $a \in \mathrm{Pos}(\mathbb{C}^{d_A})$ and $b \in \mathrm{Pos}(\mathbb{C}^{d_B})$. If $\eta > 0$ and

$$\eta \cdot \left(\frac{\max_k G_{kk}}{\sum_{j=1}^n G_{jj}} - \frac{1}{n} \right) \leq \max_{i \neq j} \frac{|G_{ij}|}{\sqrt{G_{ii} G_{jj}}}$$

for all $a \in \mathrm{Pos}(\mathbb{C}^{d_A})$ and $b \in \mathrm{Pos}(\mathbb{C}^{d_B})$ such that $G_{ii} > 0$ for all $i \in \{1, \dots, n\}$, then η satisfies the *nonlocality constraint for S*.

Information Gain / Disturbance Tradeoff



Information gain:

$$\varepsilon = \max_k p(\psi_k|m) - \frac{1}{n}$$

Disturbance:

$$\delta = \max_{i \neq j} |\langle\phi_i|\phi_j\rangle|$$

$$\eta\varepsilon \leq \delta$$

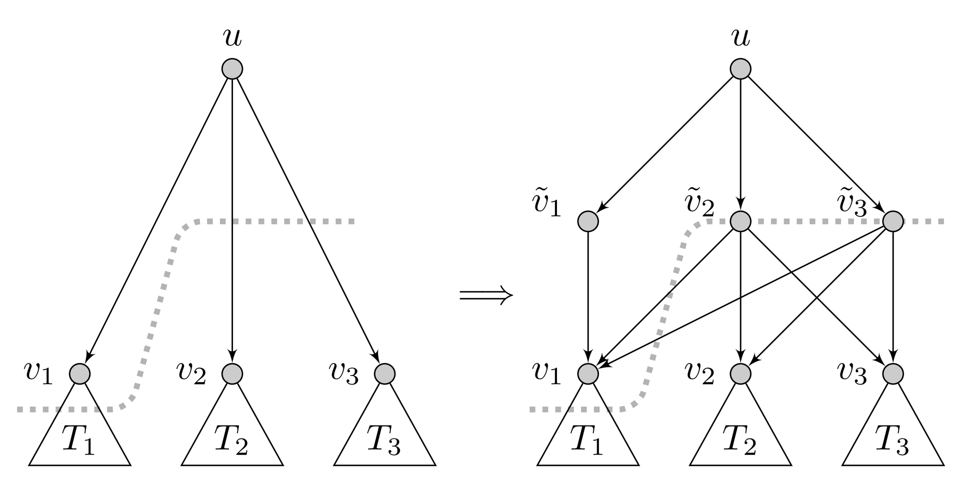
Main Result

Theorem. Let $S \subset \mathbb{C}^{d_A} \otimes \mathbb{C}^{d_B}$ be a set of n quantum states. If η satisfies the nonlocality constraint for S, then any LOCC protocol for discriminating states from S errs with probability

$$p_{\mathrm{error}} \geq \frac{2}{27} \frac{\eta^2}{n^5}$$

Proof Idea

1. Modify the original protocol so that the information gain is exactly ε (see [2]):



2. If information gain is ε, we can find two distinct post-measurement states $|\phi_i\rangle$ and $|\phi_j\rangle$ with overlap $\delta \geq \eta\varepsilon$
3. Lower bound the error probability using Helstrom's bound

Applications

- Domino states:

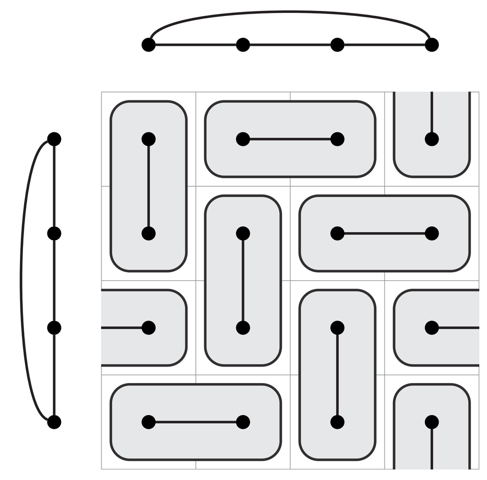
$$\eta = \frac{1}{8} \qquad p_{\mathrm{error}} = 1.96 \times 10^{-8}$$

- θ-rotated domino states:

$$\eta = \frac{\sin 2\theta}{227} \qquad p_{\mathrm{error}} = 2.43 \times 10^{-11} \sin^2(2\theta)$$

- Domino-type states:

$$\eta = \frac{1}{4D} \qquad p_{\mathrm{error}} = \frac{1}{216 D^2 (d_A d_B)^5}$$



Definition. An orthonormal product basis $S \subset \mathbb{C}^{d_A} \otimes \mathbb{C}^{d_B}$ is *domino-type* if its tiling is *irreducible* and contains only tiles of size one and two.

Open Problems

- Devise as generic method as possible for finding an η satisfying the nonlocality constraint.
- Find more applications of our framework. In particular, for cases when S is not a complete basis.
- Can our framework always be used to obtain a lower bound on p_{error} whenever such bound exists?
- Prove stronger bounds on error probability. In particular, is there a sequence $S_1, S_2, S_3, \dots$ of sets of product states such that $\lim_{k \to \infty} p_{\mathrm{error}}(S_k) = 1$?

References

[1] C. H. Bennett, D. P. DiVincenzo, C. A. Fuchs, T. Mor, E. Rains, P. W. Shor, J. A. Smolin, W. K. Wootters. *Phys. Rev. A*, 59:1070–1091, Feb 1999.

[2] M. Kleinmann, H. Kampermann, D. Bruß. *Phys. Rev. A*, 84:042326, Oct 2011.