The classical analogue of quantum mechanics

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Based on joint work with
Graeme Smith and
John Smolin from IBM

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Outline

The classical analogue of

1. Mixed states
2. Quantum entropy
3. Multipartite states
4. Bound entanglement and superactivation

The right attitude…

I think I can safely say that nobody understands quantum mechanics. So do not take the lecture too seriously, feeling that you really have to understand in terms of some model what I am going to describe, but just relax and enjoy it. I am going to tell you what nature behaves like. If you will simply admit that maybe she does behave like this, you will find her a delightful, entrancing thing. Do not keep saying to yourself, if you can possibly avoid it, 'But how can it be like that?' because you will get 'down the drain', into a blind alley from which nobody has yet escaped. Nobody knows how it can be like that.

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Classical-quantum interplay

Examples

- Classical / quantum walks [Sze04]
- Classical / quantum error correcting codes
- Classical / quantum rejection sampling [ORR13]
- Conditional distributions / superoperators [Lei06, LS11]
- …

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New insights

- New bound entangled states with private key
- Implications for classical key distillation protocols

Motivation

The Horodecki *Magnum Opus* [HHHH09]

The classical key agreement scenario is an elder sibling of an entanglement-distillation-like scenario. [...] The analogy has been recently explored and proved to be fruitful for establishing new phenomena in classical cryptography, and new links between privacy and entanglement theory. The connections are quite beautiful, however, they still remain not fully understood.

Previous work

Classical information theory

Secret key from common randomness by public discussion [Mau93, AC93]

Entanglement and distillation

- Classical analog of entanglement [CP02]

Quantum	Classical
$\frac{1}{\sqrt{2}}\|00\rangle + \frac{1}{\sqrt{2}}\|11\rangle$	$p_{00} = p_{11} = \frac{1}{2}$
Quantum bits	Secret classical bits
Classical bits	Public classical bits

- Classical vs. quantum key distillation [CEH$^+$07]

Negative information

- Conditional quantum entropy can be negative [HOW05]
- This has a classical analogue [OSW05]

Distributions vs. quantum states

State space

Classical	Quantum
$P_A \in \mathbb{R}^n_+$	$\|\psi\rangle_A \in \mathbb{C}^n$
$\sum_a p(a) = 1$	$\sum_a \|\psi(a)\|^2 = 1$

Correspondence

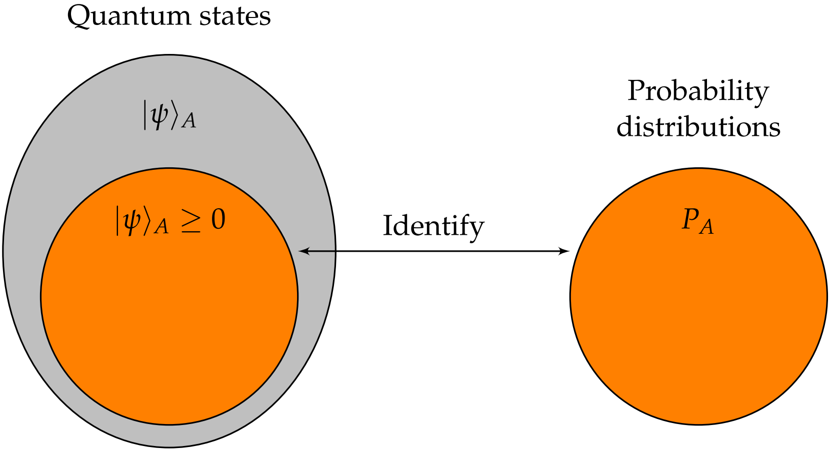
$$p(a) = |\psi(a)|^2 \qquad \left(P_A = |\psi\rangle_A^2\right)$$

$$|\psi\rangle_A = \sum_a \sqrt{p(a)}|a\rangle_A \qquad \left(|\psi\rangle_A = \sqrt{P_A}\right)$$

"Classical" quantum states

If $|\psi\rangle_A \geq 0$ we can identify $|\psi\rangle_A$ and P_A
(they are different descriptions of the same object)

The basic quantum-classical correspondence



Manifesto

1. In quantum mechanics, we never talk or think of a pure or mixed state on a given quantum system. Instead, we only use the notion of a pure state on the given system *and* a purifying system (often referred to as environment or eavesdropper). This is w.l.o.g. and is commonly referred to as the "Church of the Larger Hilbert Space".

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2. Similarly, in classical theory, we never talk or think of a probability distribution on a given state space. Instead, we always explicitly include an extra eavesdropper system and describe the joint distribution on both systems.

   Talking of probability distributions without referring to the extra eavesdropper system makes no sense!

States on a single system

$$\cancel{|\psi\rangle_A}$$

States on a single system
(and environment!)

$$|\psi\rangle_A \mapsto |\psi\rangle_A |0\rangle_E$$

Classical Schmidt decomposition

- Schmidt decomposition:

$$|\psi\rangle_{AE} = \sum_i \sqrt{\lambda_i}|\alpha_i\rangle_A|\varepsilon_i\rangle_E$$

$$\langle\alpha_i|\alpha_j\rangle = \delta_{ij} \qquad \langle\varepsilon_i|\varepsilon_j\rangle = \delta_{ij}$$

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- P_{AE} has a *classical Schmidt decomposition* if

$$|\psi\rangle_{AE} = \sqrt{P_{AE}} \quad \text{has} \quad |\alpha_i\rangle \geq 0 \quad \text{and} \quad |\varepsilon_i\rangle \geq 0$$

$$\begin{tikzcd}
P_{AE} \arrow[r, "\sqrt{\cdot}"] \arrow[d, rightsquigarrow, "\text{classical Schmidt decomposition}"'] & |\psi\rangle_{AE} \arrow[d, "\text{Schmidt decomposition}"] \\
??? & |\alpha_i\rangle_A, |\varepsilon_i\rangle_E \arrow[l, "(\,\cdot\,)^2"']
\end{tikzcd}$$

Mixed distributions

Definition

P_{AE} has a *classical Schmidt decomposition* (CSD) if

$$\sqrt{P_{AE}} = \sum_i \sqrt{\lambda_i} |\alpha_i\rangle_A |\varepsilon_i\rangle_E$$

$$|\alpha_i\rangle \geq 0 \text{ have disjoint supports}$$

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- P_{AE} is *pure* iff the sum contains one term
- "Valid" P_{AE}, $|\psi\rangle_{AE}$, and ρ_A describe the same object. It needs a new name…

quant[um]

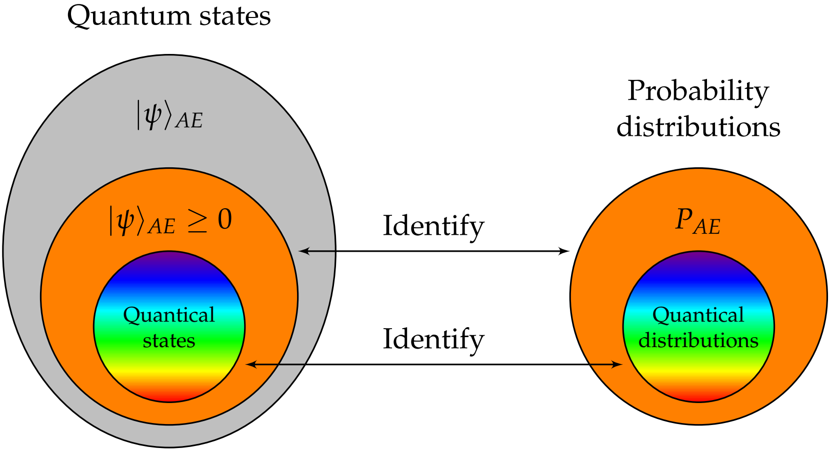
+

[class]*ical*

=

quantical

Correspondences



Examples

		E	
		0	1
A	0	$\frac{1}{2}$	
	1	$\frac{1}{2}$	

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- The quantical state space is *not* convex:

$$\frac{1}{2}\begin{bmatrix} 1 & \\ & \end{bmatrix} + \frac{1}{4}\begin{bmatrix} & \\ 1 & \end{bmatrix} + \frac{1}{4}\begin{bmatrix} & \\ & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \\ \frac{1}{4} & \frac{1}{4} \end{bmatrix}$$

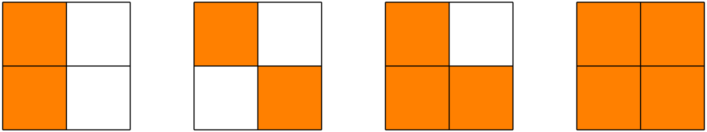
When is P_{AE} quantical?

(i) $\sqrt{P_{AE}}$ has a classical Schmidt decomposition

(ii) P_{AE} is block-diagonal:

$$P_{AE} = \begin{pmatrix} \Lambda_1 & 0 & \cdots & 0 \\ 0 & \Lambda_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \Lambda_m \end{pmatrix}$$

where $\Lambda_i = u_i \cdot v_i^{\mathsf{T}}$ for some column vectors $u_i, v_i > 0$



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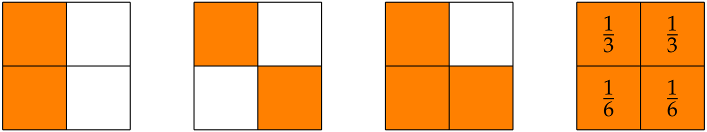

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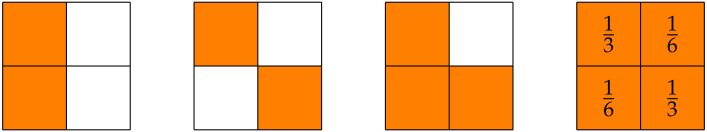

Entropy

Quantical entropy

Computing entropy from purification

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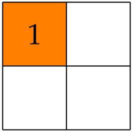
- Claim: $H(P_{AE}) = S(\rho_A)$

Quiz!



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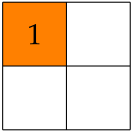

$$H(1) = 0$$

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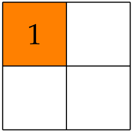

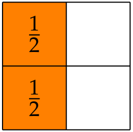

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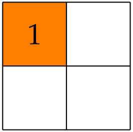

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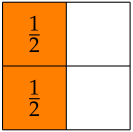

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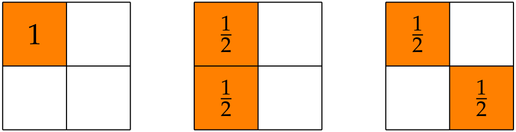

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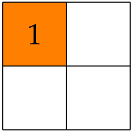

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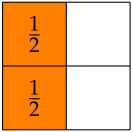

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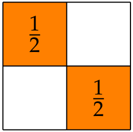
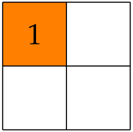
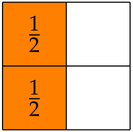
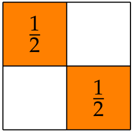


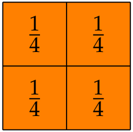
$H\left(\frac{1}{2}, \frac{1}{2}\right) = 1$

Quiz!









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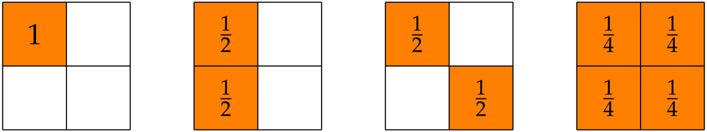

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Quiz!

1	

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$\frac{1}{2}$	

$H(1) = 0$

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	$\frac{1}{2}$

$H\left(\frac{1}{2}, \frac{1}{2}\right) = 1$

$\frac{1}{4}$	$\frac{1}{4}$
$\frac{1}{4}$	$\frac{1}{4}$

$H(1) = 0$

$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$
$\frac{1}{8}$	0	$\frac{1}{8}$
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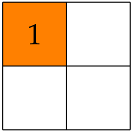

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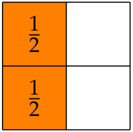

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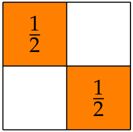

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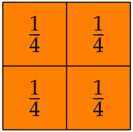
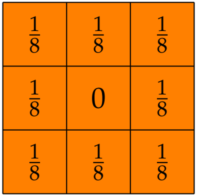


$H(1) = 0$

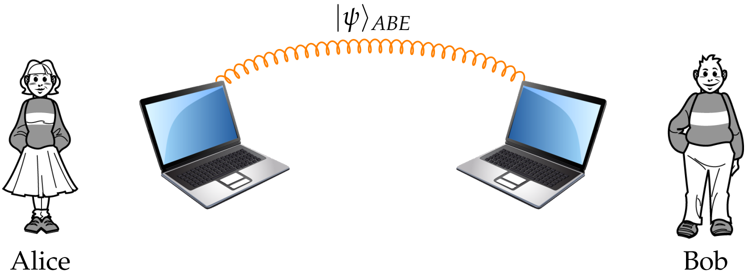


Gotcha!

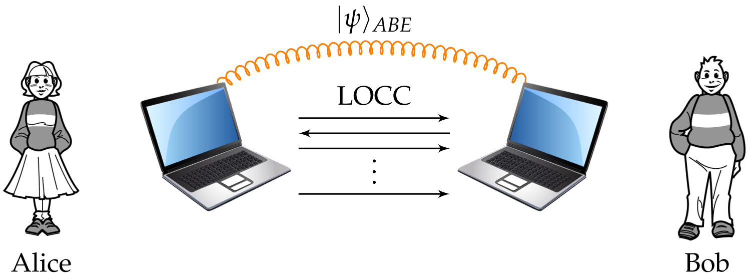
Multipartite states

$\left(|\psi\rangle_{ABE} \text{ and } P_{ABE}\right)$

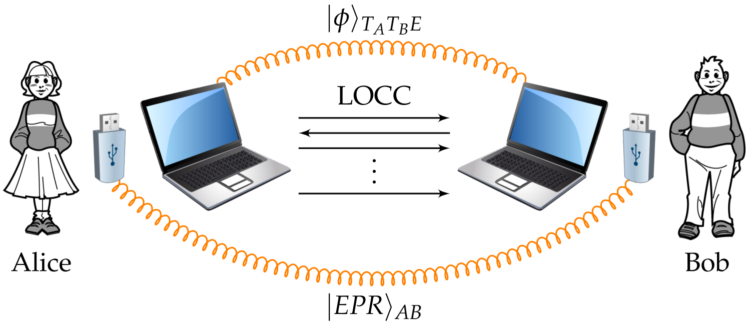
Distillation with remanent devices



Distillation with remanent devices



Distillation with remanent devices



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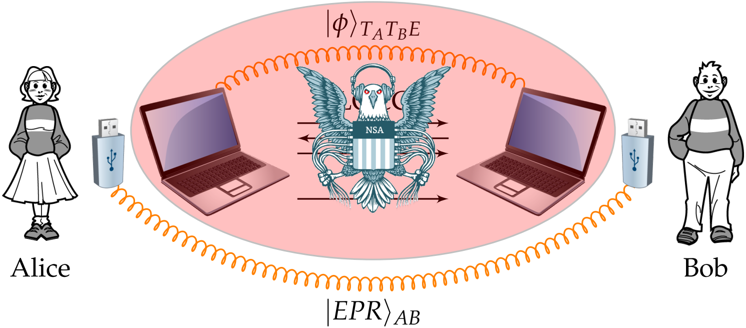
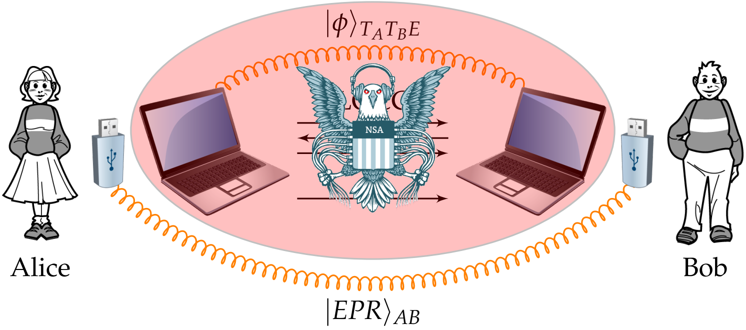
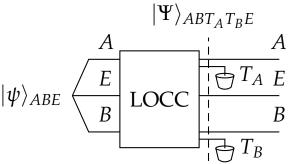


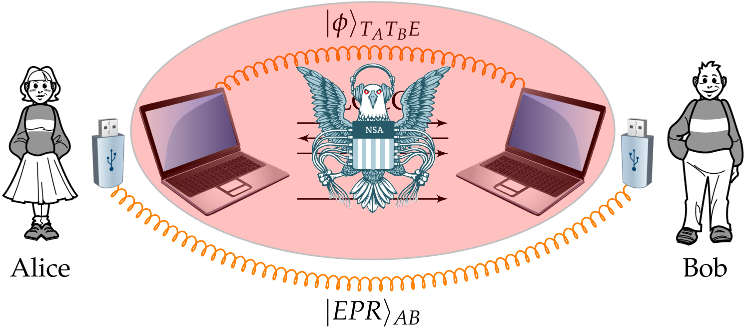
Distillation with remanent devices

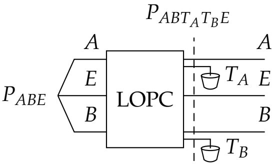




If A and B can distill an EPR pair, their key is safe even if E can also access the trash systems T_A and T_B

Distillation with remanent devices





If A and B can distill an EPR pair, their key is safe even if E can also access the trash systems T_A and T_B

cl[assical] *en*[t]*anglement*

=

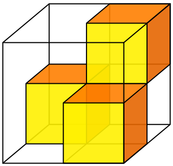
enclanglement

Unambiguous tripartite distributions

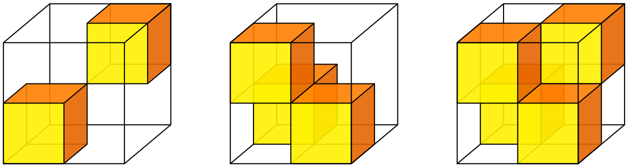
Olive property

P_{ABE} is *unambiguous* if any single party's state can be unambiguously determined by the rest of the parties

$$\forall b,e : |\{a : p(a,b,e) \neq 0\}| \leq 1$$
$$\forall a,b : |\{e : p(a,b,e) \neq 0\}| \leq 1$$
$$\forall a,e : |\{b : p(a,b,e) \neq 0\}| \leq 1$$



Genuine tripartite enclanglement



GHZ	W	Odd
000	100	100
111	010	010
	001	001
		111

Three qubits can be entangled only in two ways [DVC00] because $|GHZ\rangle$ and $|Odd\rangle$ are equivalent via $H^{\otimes 3}$

Private bound entanglement and superactivation

Bound entanglement with private key

Bound entanglement

- **Task:** Distill EPR pairs from a mixed state by LOCC
- *Bound entangled states* require entanglement to make, but no EPR pairs can be distilled from them by LOCC
- Example: PPT states

Bound entanglement with private key

Bound entanglement

- **Task:** Distill EPR pairs from a mixed state by LOCC
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Private key

- **Task:** Distill private random bits by LOCC
- Possible strategy: distill EPR pairs and measure them
- Sometimes key can be extracted even when no EPR pairs can be distilled [HHHO05, HPHH08]
- We call this phenomenon *private bound entanglement*

Classical analogue

Bound enclanglement

- **Task:** Distill private key by two-way *public discussion* from a quantical distribution (this includes error correction and privacy amplification)
- Public discussion preserves quanticality
- Key cannot be distilled from a quantical PPT distribution (otherwise EPR pairs could be distilled by LOCC)

Classical analogue

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Private key

- **Task:** Distill private key by public discussion followed by *local noisy processing* (i.e., erasing trash registers)
- **Rate:** $$K(P_{ABE}) = \max_{A\to X\to M} \left[I(X;B|M) - I(X;E|M) \right]$$ $$\geq \max_{A\to X} \left[I(X;B) - I(X;E) \right]$$

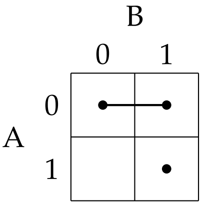
Correspondence

	Quantum	*Classical*
Unambiguous states	$\|\psi\rangle_{ABE}$	P_{ABE}
Entanglement distillation (public trash)	$D(\psi_{ABE})$	$K_{\mathrm{PD}}(P_{ABE})$
Private key distillation (private trash)	$K(\psi_{ABE})$	$K(P_{ABE})$

Theorem

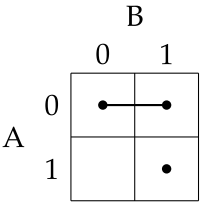
1. If $|\psi\rangle_{ABE} = \sqrt{P_{ABE}}$ is unambiguous then $D(\psi_{ABE}) \geq K_{\mathrm{PD}}(P_{ABE})$ and $K(\psi_{ABE}) \geq K(P_{ABE})$
2. There exist unambiguous distributions P_{ABE} with $K_{\mathrm{PD}}(P_{ABE}) = 0$ and $K(P_{ABE}) > 0$

Recipe



$$\big(a|00\rangle_{AB} + b|01\rangle_{AB}\big)\,|x\rangle_E$$
$$+\, c|11\rangle_{AB}|y\rangle_E$$

Recipe



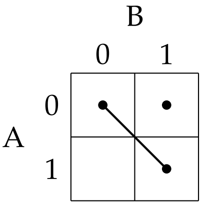
$$\big(a|00\rangle_{AB} + b|01\rangle_{AB}\big)\,|x\rangle_E$$

$$+\,c|11\rangle_{AB}|y\rangle_E$$

Unambiguous

- Union of disjoint cliques
- No repeated rows or columns within a clique

Recipe



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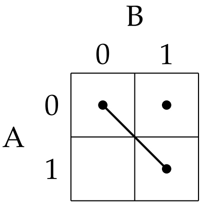

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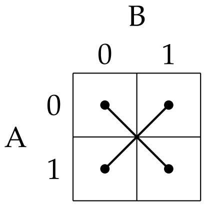
PT-invariant

- Union of crosses
- Each cross has zero determinant

Recipe



$$\left(a|00\rangle_{AB} + b|11\rangle_{AB}\right)|x\rangle_E$$
$$+\left(c|01\rangle_{AB} + d|10\rangle_{AB}\right)|y\rangle_E$$

Unambiguous

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- No repeated rows or columns within a clique

PT-invariant

- Union of crosses
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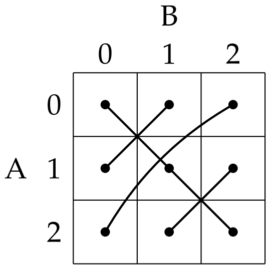
Example in 3×3

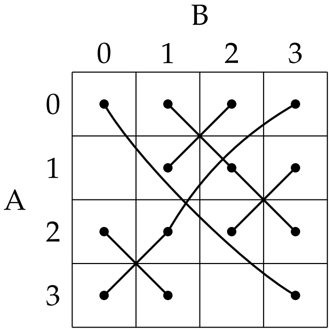


$$K(P_{ABE}) \geq 0.0057852$$

$$P_{AB} = \begin{pmatrix} 0.167184 & 0.171529 & 0.001243 \\ 0.089041 & 0.091355 & 0.017492 \\ 0.441714 & 0.017157 & 0.003285 \end{pmatrix}$$

$$Q_{X|A} = \begin{pmatrix} 1 & 0 & 0.670965 \\ 0 & 1 & 0.329035 \end{pmatrix}$$

Example in 4 × 4

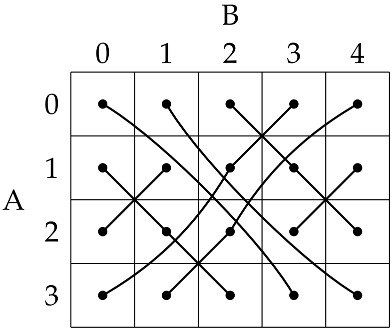


$$K(P_{ABE}) \geq 0.0293914$$

$$K(P_{ABE}) \geq 0.0213399$$

in [HPHH08]

Example in 4×5



$$K(P_{ABE}) \geq 0.0480494$$

Superactivation

Superactivation [SY08]

- Let $\mathcal{N}$ have bound entangled Choi matrix with private key
- Let $\mathcal{E}$ be the 50% erasure channel
- $Q(\mathcal{N}) = Q(\mathcal{E}) = 0$
- $Q(\mathcal{N} \otimes \mathcal{E}) \geq \frac{1}{2}P(\mathcal{N})$
- This holds also for the quantical capacities!

Summary

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Summary

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Richard Feynman: *I think I can safely say that nobody understands quantum mechanics*

This work: *To fully understand something quantum, one has to at least understand its quantical equivalent*

Open questions

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4. Is there *quantical* NPT bound enclanglement?
5. Is the optimal protocol for distilling entanglement or key from a quantical state also quantical?
6. Does quantical theory add anything to the ontic [PBR12] vs. epistemic [Spe07] debate?

Thank you!

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Capacities

Quantum capacity

$$Q(\mathcal{N}) \geq \max_{|\psi\rangle_{AA'}} \frac{1}{2} \left[I(A;B) - I(A;E) \right]_{|\phi\rangle_{ABE}}$$

where $\mathcal{N}^{A' \to BE}$ and $|\phi\rangle_{ABE} = U_{\mathcal{N}} |\psi\rangle_{AA'}$

Private capacity

$$P(\mathcal{N}) \geq \max_{\rho_{XA'}} \left[I(X;B) - I(X;E) \right]_{\sigma_{XBE}}$$

where $\rho_{XA'} = \sum_x p_x |x\rangle\langle x|_X \otimes \rho_x^{A'}$ and $\sigma_{XBE} = U_{\mathcal{N}} \rho_{XA'} U_{\mathcal{N}}^\dagger$