AUTOOSCILLATIONS IN DIELECTRIC SUSPENSION WITH A "NEGATIVE" VISCOSITY EFFECT

M. Ozols, A. Cēbers

Institute of Physics, University of Latvia,
32 Miera, LV-2169 Salaspils, Latvia (marozols@yahoo.com, aceb@tesla.sal.lv)

1. Introduction. Active systems have attracted much interest recently. They are responsible for the functioning of the cell, motility of different microorganisms [1]. Different unusual properties of these systems are known, for example, muscle cells of oysters develop high tension keeping them closed by expenditure of energy. Creation of different systems (gels and other) exhibiting the properties of active systems has been started in different labs recently [2]. Here we are exploring the properties of an active system – a suspension of dielectric particles in the liquid of low conductivity, in which the external energy is supplied by an electric field [3]. It should be stressed that electrostatic rotary machines are used by bacteria to sustain their motility [1].

It is shown that the dielectric suspension has several properties typical of the active living systems - the possibility to sustain the stretched quiescent state, the autooscillations, which are observed for insect muscles [4] and others. The physical system considered consists of a dielectric suspension with internal rotations, which model, for example, the action of the molecular motors, between two plates, one of which is free to move and connected to a spring. This mimics the thin filaments of the muscle cells, where titin and nebulin rulers serve as elastic springs in the sarcomere [1]. We have shown that depending on the physical parameters the system exhibits different regimes of autooscillations. In some range of the parameters stressed the steady state sustained by the internal rotations is unstable and autooscillations arise thus imitating the behavior of the muscle cells.

2. Model. The polarization relaxation equation is given by

$$\frac{\mathrm{d}\mathbf{P}}{\mathrm{d}t} = [\mathbf{\Omega} \times \mathbf{P}] - \frac{1}{\tau}(\mathbf{P} - \chi\mathbf{E}), \tag{1}$$

where $\mathbf{\Omega}$ is the angular velocity of a rotating particle, τ is the Maxwell relaxation time and $\chi = \chi_0 - \chi_\infty$, where χ_0 and χ_∞ are the susceptibilities of suspension polarization at low and high electric field frequencies, correspondingly. Neglecting an inertia of the small rotating particle, the balance of viscous and electrical torques gives

$$\alpha(\mathbf{\Omega} - \mathbf{\Omega}_0) = [\mathbf{P} \times \mathbf{E}], \tag{2}$$

where $\mathbf{\Omega}_0$ is the vorticity of a macroscopic flow and α is the rotational friction coefficient of the particles per unit volume. Neglecting an inertia of the free plate, the force balance on the plate along the x-axis reads

$$-\eta\frac{S}{h}\frac{\mathrm{d}x}{\mathrm{d}t} - \frac{S}{2}[\mathbf{P} \times \mathbf{E}] \cdot \mathbf{e}_z - kx = 0, \tag{3}$$

where k is a spring constant, η is the viscosity of the liquid, S is the area of the plate and h denotes the thickness of the liquid layer. The vorticity of the flow $\mathbf{\Omega}_0$ in an assumption of Couetta flow can be expressed as

$$\mathbf{\Omega}_0 = -\frac{1}{2h}\frac{\mathrm{d}x}{\mathrm{d}t}\mathbf{e}_z. \tag{4}$$

From $\mathbf{\Omega} = \Omega \mathbf{e}_z$, $\mathbf{E} = E\mathbf{e}_y$ and $\mathbf{P} = P_x\mathbf{e}_x + P_y\mathbf{e}_y$ we get $[\mathbf{P}\times\mathbf{E}] = EP_x\mathbf{e}_z$ and $[\mathbf{\Omega}\times\mathbf{P}] = -\Omega P_y\mathbf{e}_x + \Omega P_x\mathbf{e}_y$, thus. by excluding Ω from (1, 2, 3, 4) we obtain a set of equations

$$\begin{cases} \dfrac{\eta S}{kh}\dfrac{\mathrm{d}x}{\mathrm{d}t} = -\dfrac{SE}{2k}P_x - x \\ \dfrac{\mathrm{d}P_x}{\mathrm{d}t} = \dfrac{1}{2h}\dfrac{\mathrm{d}x}{\mathrm{d}t}P_y - \dfrac{E}{\alpha}P_xP_y - \dfrac{1}{\tau}P_x \\ \dfrac{\mathrm{d}P_y}{\mathrm{d}t} = \dfrac{E}{\alpha}P_x^2 - \dfrac{1}{\tau}P_y - \dfrac{1}{2h}\dfrac{\mathrm{d}x}{\mathrm{d}t}P_x + \dfrac{\chi}{\tau}E. \end{cases} \tag{5}$$

For spontaneous rotation of particles to take place, the strength of an external electric field must satisfy the condition $E > E_c$, where $E_c^2 = -\alpha/\chi\tau$ (of course ,$\chi < 0$ is necessary). A characteristic relaxation time of the plate is $\tau_p = \eta S/kh$. By substituting $t = \tau_p\mathrm{t}$ in (5), the plate relaxation time τ_p is introduced as a time scale. Similarly, by substituting $x = 2\mathrm{x}h\tau_p/\tau$ and $P_i = \chi E\mathrm{P}_i$, we obtain the following dimensionless system of differential equations:

$$\begin{cases} \dfrac{\mathrm{dx}}{\mathrm{dt}} = -\mathrm{x} + ae\mathrm{P}_x \\ \dfrac{\tau}{\tau_p}\dfrac{\mathrm{dP}_x}{\mathrm{dt}} = \dfrac{\mathrm{dx}}{\mathrm{dt}}\mathrm{P}_y + e\mathrm{P}_x\mathrm{P}_y - \mathrm{P}_x \\ \dfrac{\tau}{\tau_p}\dfrac{\mathrm{dP}_y}{\mathrm{dt}} = -\dfrac{\mathrm{dx}}{\mathrm{dt}}\mathrm{P}_x - e\mathrm{P}_x^2 - \mathrm{P}_y + 1, \end{cases} \tag{6}$$

where the parameters e and a are expressed as $e = E^2/E_c^2$ and $a = \alpha/4\eta$.

3. Autooscillations. Let us examine the case, when the Maxwell relaxation time τ for a particle is much smaller than the characteristic plate relaxation time τ_p, i.e., $\tau/\tau_p \to 0$. Thus from (6) we obtain an algebraic set of equations

$$\begin{cases} \mathrm{v} = -\mathrm{x} + ae\mathrm{P}_x \\ \mathrm{v}\mathrm{P}_y + e\mathrm{P}_x\mathrm{P}_y - \mathrm{P}_x = 0 \\ \mathrm{v}\mathrm{P}_x + e\mathrm{P}_x^2 + \mathrm{P}_y = 1, \end{cases} \tag{7}$$

where $\mathrm{v} = \mathrm{dx}/\mathrm{dt}$. By excluding v from (7), one can find that the components of particle's polarization vector satisfies $\mathrm{P}_x^2 + \mathrm{P}_y^2 = \mathrm{P}_y$.

Excluding P_x and P_y from (7) gives a force (x) and velocity (v) relationship for the active system:

$$(\mathrm{v}+\mathrm{x})^3 + 2a\mathrm{v}(\mathrm{v}+\mathrm{x})^2 + a^2(\mathrm{v}+\mathrm{x})(1 - e + \mathrm{v}^2) - a^3e\mathrm{v} = 0. \tag{8}$$

Let us denote the left side of (8) by $F(\mathrm{x},\mathrm{v})$. Since $F(0,\mathrm{v}) = 0$ has two nontrivial roots $\mathrm{v}_{1,2} = \pm a\sqrt{e(a+1)-1}/(a+1)$, the plot of implicit function (8) crosses the abscissa axis three times when $e > 1/(a+1)$, as shown in Fig. 1*a*, where a black curve contains real roots of equation (8), but gray dots show real parts of complex roots. Positive sloping of x(v) dependence corresponds to a negative friction coefficient of the active system [5]. Similarly, $F(\mathrm{x},0) = 0$ also has two nontrivial roots $\mathrm{x}_{1,2} = \pm a\sqrt{e-1}$, thus the plot of (8) crosses the ordinate axis three times when $e > 1$, as shown in Fig. 1*b*. These nontrivial roots correspond to a force created by spontaneous rotations of the particles at $e > 1$. It is interesting that the differential friction coefficient for these states in some range of the parameters explored below is negative. This causes the autooscilations around the stressed state.

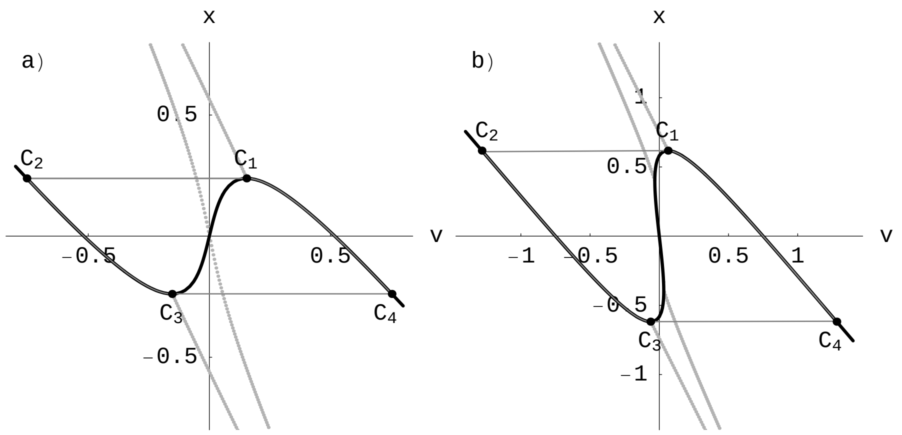

Fig. 1. Phase portraits of (8), where $a = 1.3$ and (*a*) $e = 0.8$; (*b*) $e = 1.2$.

As one can see from Fig. 1, periodic autooscillations take place when there is a jump from point C_1 in the phase plane to C_2, where the coordinate is the same, but the velocity of the oscillating plane is opposite. Similarly, a jump from C_3 to C_4 happens, thus a closed cycle $\mathrm{C}_1, \mathrm{C}_2, \mathrm{C}_3, \mathrm{C}_4$ is formed up. A characteristic shape of autooscillations is shown in Fig. 2.

Now let us find all values of the parameters e and a, for which such periodic behavior can be observed. For the jump from C_1 to C_2 to happen, it is necessary that the maximum of function $\mathrm{x(v)}$, i.e., point C_1, lies in the first quadrant, otherwise there will be a stable stationary point on the x-axis, which cannot be crossed. Thus we have to solve $\partial F\big(\mathrm{x(v)}, \mathrm{v}\big)/\partial \mathrm{v} = 0$ together with $F\big(\mathrm{x(v)}, \mathrm{v}\big) = 0$ with restrictions $\mathrm{x} > 0$ and $\mathrm{v} > 0$. This leads to inequalities

$$\frac{1}{a+1} < e < 2\frac{a+1}{a+2}, \tag{9}$$

where $a > 0$. Indeed, as one can see from graphs similar to Fig. 1, the periodic behavior cannot observed in either cases: a) if e is too small, i.e., $e < 1/(a+1)$ or b) if e is too big, i.e., $e > 2(a+1)/(a+2)$.

To define the period of autooscillations, we consider the symmetry of plot of the implicit function $F(\mathrm{x}, \mathrm{v}) = 0$ and that the jump from C_1 to C_2 happens momentarily (of course, inequalities (9) must be satisfied for the jump to occur). By perceiving v as a function of x in the segment $\mathrm{C}_4\mathrm{C}_1$, one can find the period

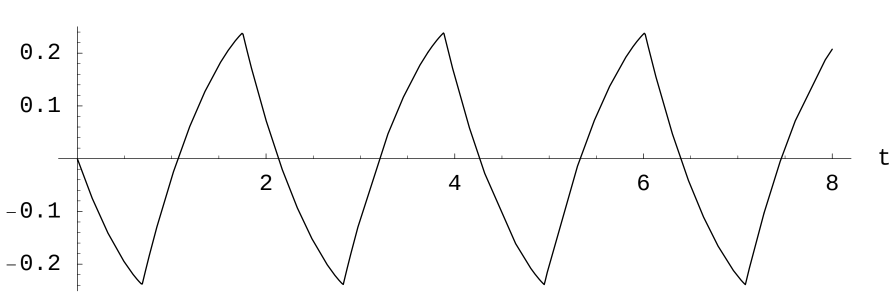

Fig. 2. Shape of oscillations $\mathrm{x(t)}$, where $a = 1.3$ and $e = 0.8$.

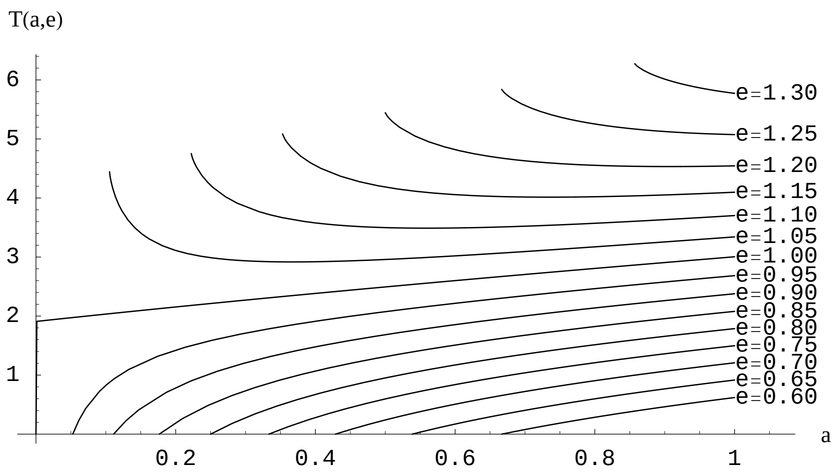

Fig. 3. Period dependence on a.

by integrating dx/v(x), i.e.,

$$T = 2 \int_{-\mathrm{x}_c}^{+\mathrm{x}_c} \frac{\mathrm{dx}}{\mathrm{v(x)}}$$

where x_c is the ordinate of the critical point C_1. Period dependence on the parameters is shown in Fig. 3.

4. Conclusions. We have illustrated here that the dielectric suspension in the electric field behaves similarly to the active systems of the living world. The obtained results can be useful forcreating the artificial active systems for microfluidics and others.

REFERENCES

1. D. Bray. *Cell Movements*, Garland Publishing (2001).
2. R. Voituriez, J.F. Joanny, J. Prost. *arXiv:q-bio.SC/0503022* 16 Mar (2005).
3. A. Cebers. *Phys. Rev. Lett.*, vol. 92 (2004), p. 034501.
4. F. Julicher, J. Prost. *Phys.Rev.Lett.*, vol. 78 (1997), p. 4510.
5. A. Cebers. *Magnetohydrodynamics*, vol. 16 (1980), p. 175.