Ticamības intervāla konstruēšana ar drošību $\beta = 1 - \alpha.$

problēma	statistika	intervāls
$\xi \sim N(\theta, \sigma^2)$ $J_{\theta,\beta} = ?$	$T(x) = \frac{\overline{x} - \theta}{\sigma}\sqrt{n} \sim N(0,1)$	$P(\lvert T(x) \rvert < t) = \beta$ $J_{\theta,1-\alpha} = (\overline{x} - t\frac{\sigma}{\sqrt{n}}, \overline{x} + t\frac{\sigma}{\sqrt{n}})$
ξ ar $F_\xi(x,\theta)$ $(n-\text{liels})$ $a)\ b(1,\theta)$ $b)\ P(\theta)$ $c) eks(\theta)$ $d)\ \chi^2_\theta$	$a) T(x) = \frac{\overline{x} - \theta}{\sqrt{\theta(1-\theta)}}\sqrt{n}$ $b) T(x) = \frac{\overline{x} - \theta}{\sqrt{\theta}}\sqrt{n}$ $c) T(x) = \frac{\overline{x} - 1/\theta}{1/\theta}\sqrt{n}$ $d) T(x) = \frac{\overline{x} - \theta}{\sqrt{2\theta}}\sqrt{n}$	$P(\lvert T(x) \rvert < t) = \beta$ $T(x) \approx N(0,1)$ $\Downarrow$ $J_{\theta,\beta}$

$\xi \sim N(\theta_1, \theta_2)$ $a)\ J_{\theta_1,\beta} = ?$ $b) J_{\theta_2,\beta} = ?$	$a) T(x) = \frac{\overline{x} - \theta_1}{S}\sqrt{n-1} = t_{n-1}$ $b) n < 20 \Rightarrow \frac{S^2 n}{\theta_2} = \chi^2_{n-1}$ $n \geq 20 \Rightarrow$ $T_1(x) = \frac{\frac{S^2 n}{\theta_2} - (n-1)}{\sqrt{2(n-1)}} \approx N(0,1)$	$a) n < 20 \Rightarrow T(x) \sim$ Stjūdenta sad, $n \geq 20 \Rightarrow T(x) \sim N(0,1)$ $P(\lvert T(x)\rvert < t) = \beta,$ $J_{\theta,\beta} = (\overline{x} - t\frac{S}{\sqrt{n-1}}, \overline{x} + t\frac{S}{\sqrt{n-1}})$ $b)\begin{cases} P(\chi^2_{n-1} > l_2) = \frac{1-\beta}{2} \\ P(\chi^2_{n-1} < l_1) = \frac{1-\beta}{2} \end{cases}$ $n < 20 \Rightarrow J_{\theta,\beta} = (\frac{S^2 n}{l_2}, \frac{S^2 n}{l_1})$ $n \geq 20 \ \rightarrow\ P(\lvert T_1(x)\rvert < t) = \beta \Rightarrow$ $\left(\frac{S^2 n}{n-1+t\sqrt{2(n-1)}}, \frac{S^2 n}{n-1-t\sqrt{2(n-1)}}\right)$

$\xi \sim N(\theta_1, \sigma_1^2),$ $\eta \sim N(\theta_2, \sigma_2^2)$ neatkarīgi $J_{\theta_1-\theta_2,\beta} = ?$	$T(x,y) = \dfrac{\overline{x} - \overline{y} - (\theta_1 - \theta_2)}{\sqrt{\dfrac{\sigma_1^2}{n} + \dfrac{\sigma_2^2}{m}}}$	$T(x,y) \sim N(0,1)$ $P(\lvert T(x,y) \rvert < t) = \beta$ $\Downarrow$ $J_{\theta_1-\theta_2,\beta}$
$\xi \sim N(\theta_1, \theta),$ $\eta \sim N(\theta_2, \theta)$ neatkarīgi $J_{\theta_1-\theta_2,\beta} = ?$	$T(x,y) =$ $= \dfrac{\overline{x} - \overline{y} - (\theta_1 - \theta_2)}{\sqrt{\dfrac{1}{n} + \dfrac{1}{m}}\sqrt{S_x^2 n + S_y^2 m}} \sqrt{n+m-2} =$ $= t_{n+m-2}$	$T(x,y) = t_{n+m-2}$ $P(\lvert T(x,y) \rvert < t) = \beta$ $\Downarrow J_{\theta_1-\theta_2,\beta}$

$(\xi,\eta),\ F_{(\xi,\eta)}(x,y)$ $\rho(\xi,\eta)=$ $\dfrac{E(\xi-E\xi)(\eta-E\eta)}{\sigma_\xi\sigma_\eta}$ $J_{\rho(\xi,\eta),\beta}=?$	$\frac{1}{2}\ln\frac{1+r}{1-r}\approx N(\frac{1}{2}\ln\frac{1+\rho}{1-\rho},\frac{1}{n-3})$ $T(x,y)=$ $(\ln\frac{1+r}{1-r}-\ln\frac{1+\rho}{1-\rho})\frac{\sqrt{n-3}}{2}\approx N(0,1)$ $r_{\xi,\eta}=\dfrac{\dfrac{\sum_{i=1}^{n}x_iy_i}{n}-\overline{x}\,\overline{y}}{S_xS_y}$	$P(\lvert T(x,y)\rvert<t)=\beta$ $T(x,y)\sim N(0,1)$ $\Downarrow$ $J_{\rho(\xi,\eta),\beta}$
$\xi\sim N(\theta_1,\theta_2),$ $\eta\sim N(\theta_1',\theta_2')$ neatkarīgi $J_{\frac{\theta_2}{\theta_2'},\beta}=?$	$T(x,y)=\dfrac{S_x^2n\theta_2'}{S_y^2m\theta_2}=\dfrac{s_x^2\theta_2'}{s_y^2\theta_2}$	$T(x,y)=F(n-1,m-1)$ $\begin{cases}P(F(n-1,m-1)>l_2)=\dfrac{1-\beta}{2}\\ P(F(m-1,n-1)>l_2)=\dfrac{1+\beta}{2}\end{cases}$ $\Downarrow$ $J_{\frac{\theta_2}{\theta_2'},\beta}$

Statistiskā hipotēžu pārbaude.

nulles hipotēze	**statistika(kritērijs), ja H_0 spēkā**	**alternatīva**	**kritiskais apgabals**
$\xi \sim N(\theta_1, \sigma_1^2)$, $\eta \sim N(\theta_2, \sigma_2^2)$ neatkarīgi, $x = (x_1, ..., x_n)$, $y = (y_1, ..., y_m)$ $H_0 : \theta_1 = \theta_2$	$T(x,y) = \dfrac{\overline{x} - \overline{y}}{\sqrt{\dfrac{\sigma_1^2}{n} + \dfrac{\sigma_2^2}{m}}} \sim N(0,1)$	*a)* $H_1 : \theta_1 \neq \theta_2$ *b)* $H_1 : \theta_1 < \theta_2$ *c)* $H_1 : \theta_1 > \theta_2$	$P(\lvert T(x,y) \rvert > z \mid H_0) = \alpha \Rightarrow$ $S = (-\infty, -z) \bigcup (z, \infty)$ $P(T(x,y) > z \mid H_0) = \alpha \Rightarrow S = (z, \infty)$ $P(T(x,y) < z \mid H_0) = \alpha \Rightarrow S = (-\infty, z)$
$\xi \sim N(\theta_1, \theta)$, $\eta \sim N(\theta_2, \theta)$ neatkarīgi, $x = (x_1, ..., x_n)$, $y = (y_1, ..., y_m)$ $H_0 : \theta_1 = \theta_2$	$T(x,y) = \dfrac{(\overline{x} - \overline{y})(n+m-2)}{\sqrt{(\dfrac{1}{n} + \dfrac{1}{m})(S_x^2 n + S_y^2 m)}}$ $T(x,y) = t_{n+m-2}$	*a)* $H_1 : \theta_1 \neq \theta_2$ *b)* $H_1 : \theta_1 < \theta_2$ *c)* $H_1 : \theta_1 > \theta_2$	$P(\lvert T(x,y) \rvert > z \mid H_0) = \alpha \Rightarrow$ $S = (-\infty, -z) \bigcup (z, \infty)$ $P(T(x,y) > z \mid H_0) = \alpha \Rightarrow S = (z, \infty)$ $P(T(x,y) < z \mid H_0) = \alpha \Rightarrow S = (-\infty, z)$

$\xi \sim N(\theta_1, \theta_2)$, $\eta \sim N(\theta_1', \theta_2')$ neatkarīgi, $x = (x_1, ..., x_n)$, $y = (y_1, ..., y_m)$ $H_0 : \theta_2 = \theta_2'$	$T(x,y) = \frac{s_x^2}{s_y^2} = \frac{S_x^2 n(m-1)}{S_y^2 m(n-1)} = F(n-1, m-1)$	$a) H_1 : \theta_2 \neq \theta_2'$ $b) H_1 : \theta_2 < \theta_2'$	$a) \begin{cases} P(T(x,y) > t_2 \mid H_0) = \frac{\alpha}{2} \\ P(\frac{1}{T(x,y)} > \frac{1}{t_1} \mid H_0) = \frac{\alpha}{2} \end{cases} \Rightarrow$ $S = (0, t_1) \bigcup (t_2, \infty)$ $b) P(T(x,y) > t \mid H_0) = \alpha$ $\Rightarrow S = (t, \infty)$
$\xi \sim b(1, \theta)$, $\theta \in (0,1)$ $x = (x_1, ..., x_n)$ $n - liels$ $H_0 : \theta = p_0$	$T(x) = \frac{\sum_{i=1}^{n} x_i - n\theta}{\sqrt{n\theta(1-\theta)}} \approx N(0,1)$, $\forall \theta \in (0,1)$	$a)\ H_1 : \theta \neq p_0$ $b)\ H_1 : \theta > p_0$ $c)\ H_1 : \theta < p_0$	$P(\mid T(x) \mid > z \mid H_0) = \alpha \Rightarrow$ $S = (-\infty, -z) \bigcup (z, \infty)$ $P(T(x) > z \mid H_0) = \alpha \Rightarrow S = (z, \infty)$ $P(T(x) < z \mid H_0) = \alpha \Rightarrow S = (-\infty, z)$

Neparametrisko hipotēžu pārbaude.

problēma	***statistika***	***intervāls***
$H_0 : F_\xi(x) = F(x)$ $H_1 : F_\xi(x) \neq F(x)$	χ^2 ***-kritērijs*** $T(x) = \sum_{i=1}^{k} \frac{(n_i - np_i)^2}{np_i} \approx \chi^2_{k-1-l}$, k – int ervālu skaits, n_i -i-tajā intervālā variantu skaits, l -novērtējumu skaits, $p_i = P(\xi \in (x, x_{i+1}) \mid H_0), i = 1, 2..., k$	$P(T(x) > \varepsilon \mid H_0) = \alpha \Rightarrow$ $S = (\varepsilon, \infty)$
$H_0 : F_\xi(x) = F(x)$ $H_1 : F_\xi(x) \neq F(x)$	***Kolmogorova kritērijs*** $T(x) = \sqrt{n} \sup_x \mid F_n(x) - F(x) \mid \approx K$	$P(T(x) > z) = \alpha \Leftrightarrow$ $1 - K(z) = \alpha \Rightarrow$ $S = (z, \infty)$
$H_0 : F_\xi(x) = F_\eta(x)$ $H_1 : \not H_0$ $x = (x_1, x_2, ..., x_n)$ $y = (y_1, y_2, ..., y_m)$ *–neatkarīgas izlases*	***Smirnova-Kolmogorova kritērijs*** $T(x, y) = \sqrt{\frac{nm}{n+m}} \sup_x \mid F_n(x) - F_m(x) \mid \approx K$ $F_n(x), F_m(x)$ – empīriskās sadalījuma funkcijas	$P(T(x, y) > \varepsilon \mid H_0) = \alpha$ $\Rightarrow \ S = (\varepsilon, \infty)$

$H_0 : \xi, \eta - neatkarīgi$ $H_1 : \not H_0$ $(x, y) =$ $\{(x_1, y_1), (x_2, y_2), ...(x_n, y_n)\}$ *–izlase*	***Divu izlašu neatkarības kritērijs*** $T(x, y) = n\left[\sum_{j=1}^{s}\sum_{i=1}^{k}\frac{\nu_{ij}^2}{\nu_{i.}\nu_{.j}} - 1\right] \approx \chi^2_{(k-1)(s-1)}$ ν_{ij}-notikuma $\{\xi \in (x_i, x_{i+1}), y \in (y_j, y_{j+1})\}$ skaits, $i = (1, 2, ..., k), j = (1, 2, ..., s),$ $\nu_{i.} = \sum_{j=1}^{s}\nu_{\text{ij}}, \quad \nu_{.j} = \sum_{i=1}^{k}\nu_{\text{ij}}, \quad \sum_{j=1}^{j}\sum_{i=1}^{k}\nu_{ij} = n$	$P(T(x, y) > \varepsilon \mid H_0) = \alpha$ $\Rightarrow \; S = (\varepsilon, \infty)$
$H_0 : F_{\xi_1}(x) = ... = F_{\xi_k}(x)$ $H_1 : \not H_0$ $x^j = (x_1{}^j, x_2{}^j, ..., x_{n_j}{}^j)$ $j = i, ..., k -$ neatkarīgas izlases $n = \sum_{j=1}^{k} n_j$	***Izlašu līdzības kritērijs*** $T(x^1, ..., x^k) = n\left[\sum_{j=1}^{s}\sum_{i=1}^{k}\frac{\nu_{ij}^2}{\nu_{i.}\nu_{.j}} - 1\right] \approx \chi^2_{(k-1)(s-1)}$ $\nu_{ij} -$ notikuma $\{\xi_j \in (x_i, x_{i+1})\}, i = 1, ..., s$ skaits, j=1,...,k, $\sum_{i=1}^{s}\nu_{ij} = \nu_{.j} = n_j, \; \sum_{j=1}^{s}\sum_{i=1}^{k}\nu_{ij} = n, \; \sum_{j=1}^{k}\nu_{ij} = \nu_{i.}$	$P(T(x^1, ..., x^k) > \varepsilon \mid H_0)$ $= \alpha \; \Rightarrow \; S = (\varepsilon, \infty)$

$H_0 : F_\xi(x) = F_\eta(x)$ $H_1 : \not H_0$ $x = (x_1, x_2, ..., x_n)$ $y = (y_1, y_2, ..., y_m)$ *–neatkarīgas izlases*	***Vilkoksona kritērijs*** $T(x,y) = \dfrac{u - \dfrac{mn}{2}}{\sqrt{\dfrac{mn}{12}(m+n+1)}} \approx N(0,1),$ $u = \sum_{k=1}^{m} u_k$ ~ inversiju skaits, u_k ~ "notikumu $\{x_i < y_k$,i=1,...,n $\}$ skaits" $k = 1,...m, r \in \{0,1,...,n\}$	$P(\mid T(x,y) \mid > \varepsilon \mid H_0) = \alpha$ $\Rightarrow S = (-\infty, -\varepsilon) \bigcup (\varepsilon, \infty)$
$\xi \in \mathrm{X}_\xi$, ar $F_\xi(x)$ $H_0 : \max\limits_{1 \le i \le n} x_i \in \mathrm{X}_\xi$ $H_1 : \not H_0$ $x = (x_1, ..., x_n)$	***Rupjo kļūdu analīze*** $T(x) = \max\limits_{1 \le i \le n} x_i,$ $F_{T(x)}(x) = F_\xi^n(x)$	$P(T(x) > \varepsilon \mid H_0) = \alpha$ $\Leftrightarrow F_\xi(x) = \sqrt[n]{1-\alpha}$ $\Rightarrow S = (\varepsilon, \infty)$

Regresijas līknes analīze.

Ticamības intervālu konstruēšana ar drošību $1-\alpha$**.**

problēma	statistika	intervāls	tabula
$J_{\beta^*,1-\alpha}$ - lineāras regresijas koeficienta β ticamības intervāls	$T(x,y)=\frac{(b-\beta^*)S_x}{S_y\sqrt{1-r^2}}\sqrt{n-2}=t_{n-2}$ $T_1(x,y)=\frac{(b-\beta^*)^2\sum_{i=1}^{n}(x_i-\bar{x})^2}{\sum_{i=1}^{n}(Y_i-\hat{y}(x_i))^2}(n-2)=$ $=F(1,n-2)$	$P(\mid T(x,y)\mid<\varepsilon)=1-\alpha$ $\begin{cases}P(F(1,n-2)>l_2)=\frac{\alpha}{2}\\ P(F(n-2,1)>\frac{1}{l_1})=\frac{\alpha}{2}\end{cases}$	t_ν – *Stjūdenta sad.* *Fišera sad.*
$J_{\sigma^2,1-\alpha}$ $-\sigma^2$ ticamības intervāls	$T(x,y)=\frac{\sum_{i=1}^{n}(Y_i-\hat{y}(x_i))^2}{\sigma^2}=$ $=\frac{nS_y^2(1-r_{x,y}^2)}{\sigma^2}=\chi^2_{n-2}$	$\begin{cases}P(T(x,y)>\varepsilon_2)=\frac{\alpha}{2}\\ P(T(x,y)>\varepsilon_2)=1-\frac{\alpha}{2}\end{cases}$	χ^2_ν, $\nu=n-2$

$J_{\bar{y}(x),1-\alpha} - \bar{y}(x),$ $x \in$ X ticamības intervāls	$T(x,y) = \frac{(\hat{y}(x) - \bar{y}(x))\sqrt{n-2}}{\sqrt{\frac{1}{n} + \frac{(x-\bar{x})^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}}\sqrt{\sum_{i=1}^{n}(Y_i - \hat{y}(x_i))^2}} =$ $= t_{n-2},$ $\hat{y}(x) = \bar{Y} + r_{x,y}\frac{S_y}{S_x}(x - \bar{x})$	$P(\lvert T(x,y)\rvert < \varepsilon) = 1 - \delta$	$t_\nu -$ *Stjūdenta sad.*

Hipotēžu pārbaude lineārās regresijas analīzē.

hipotēzes	***statistika***	***kritiskais apgabals, kurā ar varbūtību α noraida nulles hipotēzi***
$H_0 : \beta^* = \beta_0$ $H_1 : \beta^* \neq \beta_0$	$T(x,y) = \frac{(b-\beta^*)S_x}{S_y\sqrt{1-r^2}}\sqrt{n-2} =$ $= \frac{b-\beta^*}{s.k.(b)} = t_{n-2}$	$P(\lvert T(x,y)\rvert > t_{\alpha,n-2} \mid \beta^* = \beta_0) = \alpha$ $S_{kr} = (-\infty, -t_{\alpha,n-2}) \bigcup (t_{\alpha,n-2}, \infty)$

$H_0 : \beta^* = \beta_0$ $H_1 : \beta^* > \beta_0$	$T(x,y) = \frac{b - \beta^*}{s.k.(b)} = t_{n-2}$	$P(T(x,y) > l_{\alpha,n-2} \mid \beta^* = \beta_0) = \alpha$ $S_{kr} = (l_{\alpha,n-2}, \infty)$
$H_0 : \beta^* = \beta_0$ $H_1 : \beta^* < \beta_0$	$T(x,y) = \frac{b - \beta^*}{s.k.(b)} = t_{n-2}$	$P(T(x,y) < r_{\alpha,n-2} \mid \beta^* = \beta_0) = \alpha$ $S_{kr} = (-\infty, r_{\alpha,n-2})$
$H_0 : \beta^* = 0 \Rightarrow \rho(X,Y) = 0$ $H_1 : \beta^* \neq 0$	$T(x,y) = \frac{b - \beta^*}{s.k.(b)} = t_{n-2}$	$P(\mid T(x,y) \mid > t_{\alpha,n-2} \mid \beta^* = 0) = \alpha$ $S_{kr} = (-\infty, -t_{\alpha,n-2}) \bigcup (t_{\alpha,n-2}, \infty)$