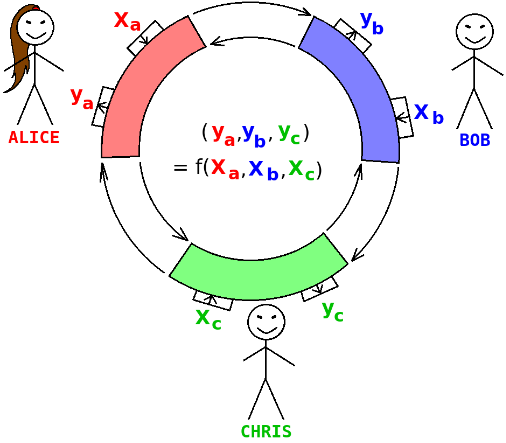
Bit Decomposition Protocols in Secure Multiparty Computation

19.10.18

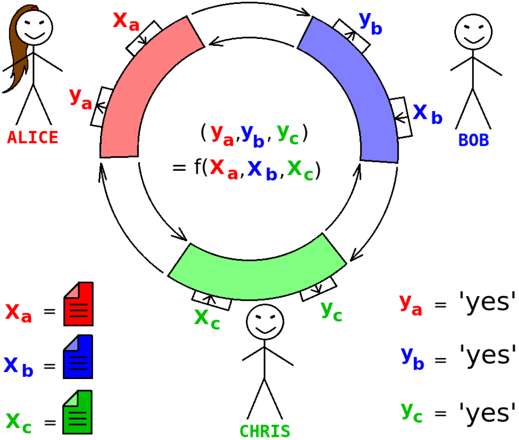
Peeter Laud Alisa Pankova

CYBERNETICA

Secure Multiparty Computation



Secure Multiparty Computation



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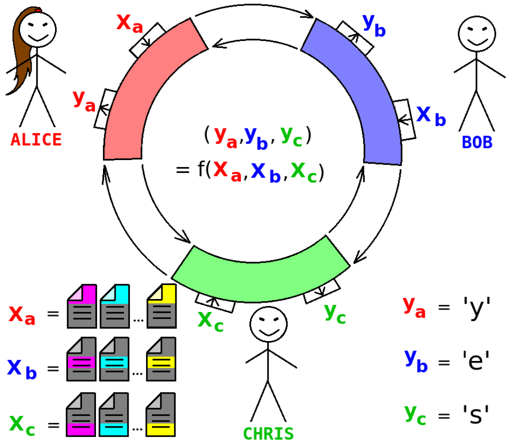

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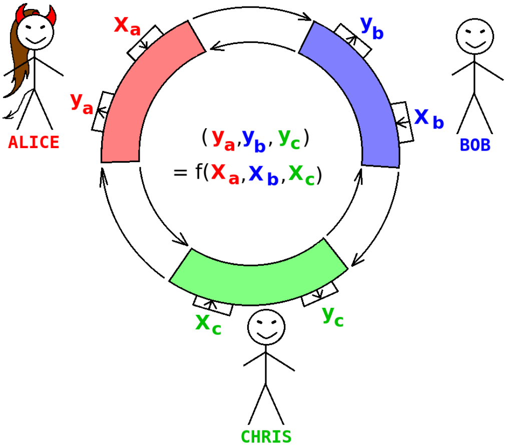

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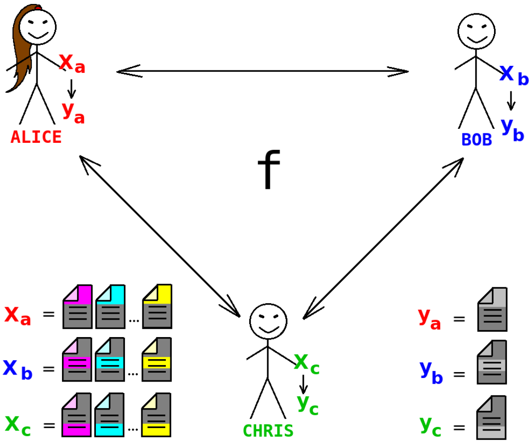
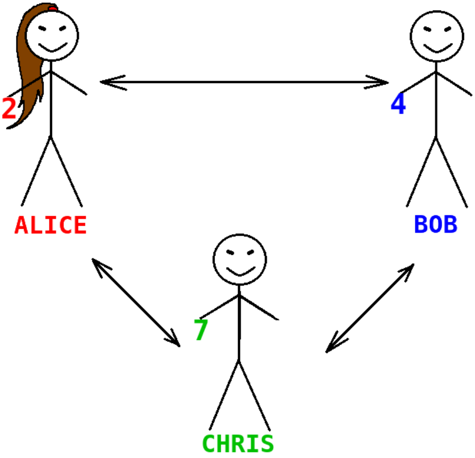


Two main types of sharing



- $x = 2 + 4 + 7 = 13 = 5 \pmod{2^3}$

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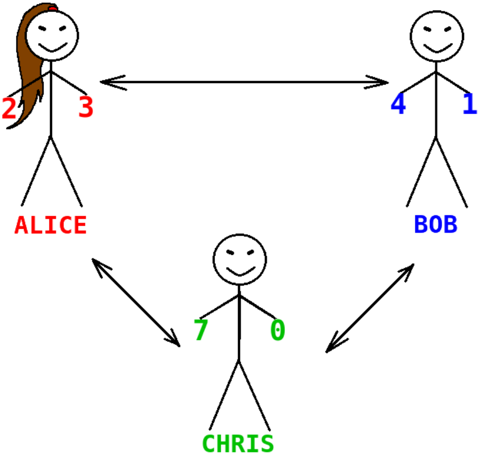

- $x = 2 + 4 + 7 = 13 = 5 \pmod{2^3}$
- $y = 3 + 1 + 0 = 4 \pmod{2^3}$

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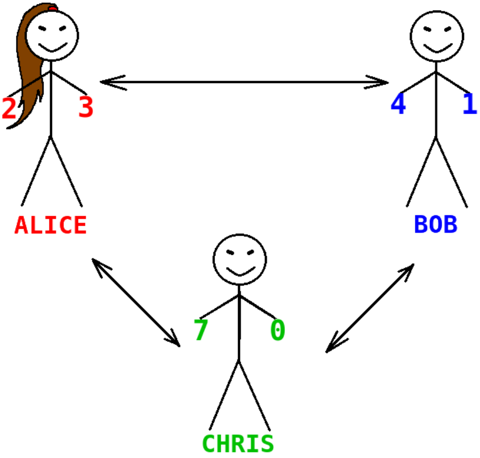

- $x = 2 + 4 + 7 = 13 = 5 \pmod{2^3}$
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- $x + y = 2 + 3 + 4 + 1 + 7 + 0 = 17 = 1 \pmod{2^3}$

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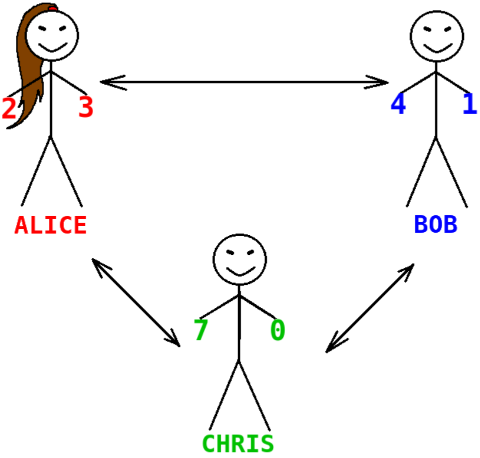

- $x = 2 + 4 + 7 = 13 = 5 \pmod{2^3}$ good for linear operations
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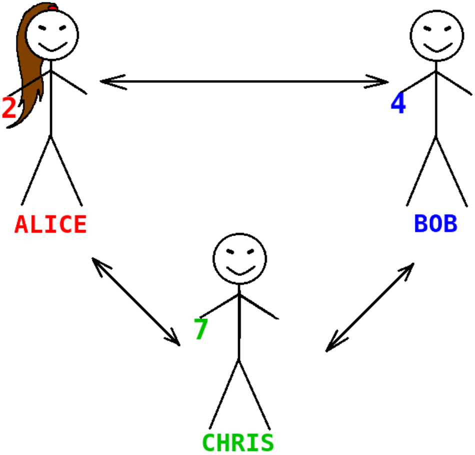

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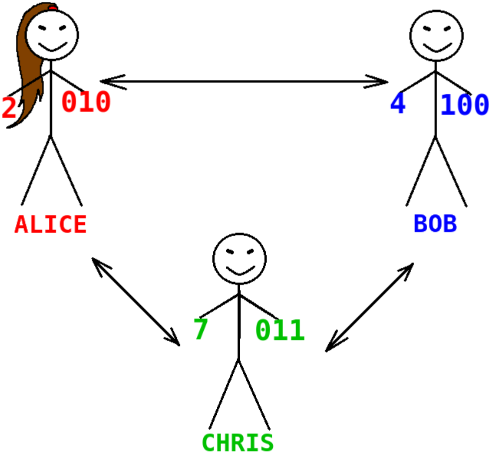
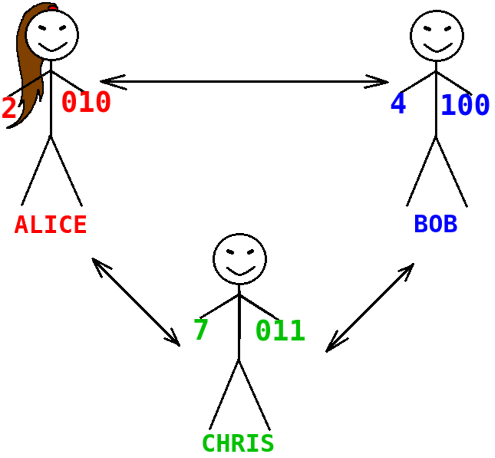


- $x = 2 + 4 + 7 = 13 = 5 \pmod{2^3}$ good for linear operations
- $x = 010 \oplus 100 \oplus 011 = 101_2$ good for bitwise operations

Two main types of sharing



- $x = 2 + 4 + 7 = 13 = 5 \pmod{2^3}$ good for linear operations
- $x = 010 \oplus 100 \oplus 011 = 101_2$ good for bitwise operations
- We need to convert between the two sharings $\mathbb{Z}_{2^n}$ and $\mathbb{Z}_2^n$.

Sharing a bit: over $\mathbb{Z}_2$ vs over $\mathbb{Z}_{2^n}$

$$\mathbb{Z}_2 : \square\,\square\,\boxed{1} + \square\,\square\,\boxed{1} = \square\,\square\,\boxed{0}$$

Sharing a bit: over $\mathbb{Z}_2$ vs over $\mathbb{Z}_{2^n}$

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Sharing a bit: over $\mathbb{Z}_2$ vs over $\mathbb{Z}_{2^n}$

$\mathbb{Z}_2$:			1	+			1	=			0
$\mathbb{Z}_{2^3}$:	0	0	1	+	0	0	1	=	0	1	0
	0	0	0	+	0	0	0	=	0	0	0
	1	0	1	+	0	1	1	=	0	0	0

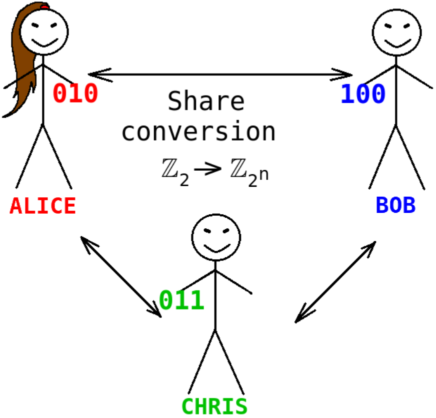
Sharing a bit: over $\mathbb{Z}_2$ vs over $\mathbb{Z}_{2^n}$

$\mathbb{Z}_2$:	1	+	1	=	0
$\mathbb{Z}_{2^3}$:	0 0 1	+	0 0 1	=	0 1 0
	0 0 0	+	0 0 0	=	0 0 0
	1 0 1	+	0 1 1	=	0 0 0

There exists Share conversion SMC protocol: $\mathbb{Z}_2 \to \mathbb{Z}_{2^n}$

- Let us use it as a black box.

The first method for $\mathbb{Z}_2^n \to \mathbb{Z}_{2^n}$: bitwise conversion



- $x = 010 \oplus 100 \oplus 011 = 101_2 = 5$

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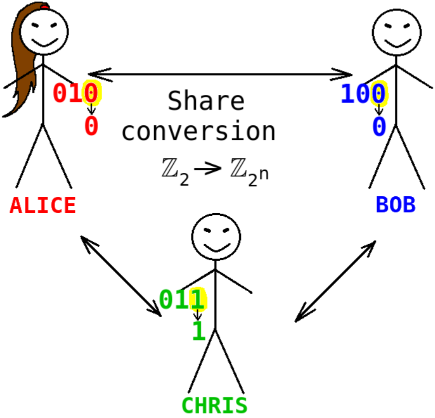

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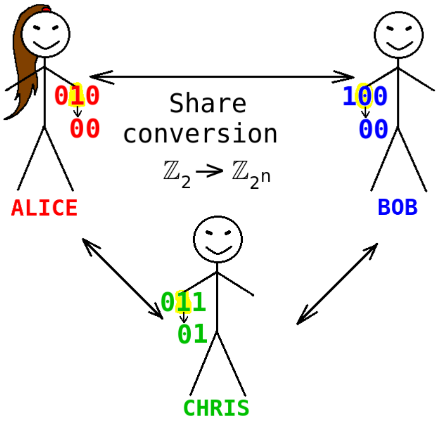

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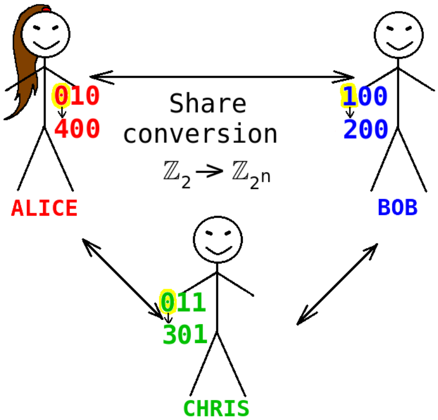

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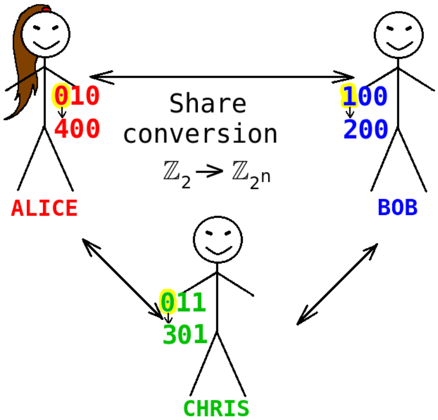
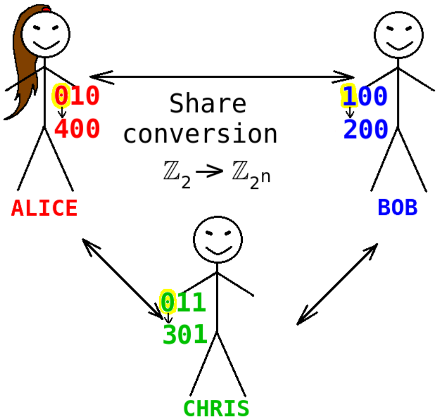


- $x = 010 \oplus 100 \oplus 011 = 101_2 = 5$
- $x = (4 \cdot 4 + 2 \cdot 0 + 0) + (4 \cdot 2 + 2 \cdot 0 + 0) + (4 \cdot 3 + 2 \cdot 0 + 1) = 16 + 8 + 12 + 1 = 37 = 5 \pmod{2^3}$

The first method for $\mathbb{Z}_2^n \to \mathbb{Z}_{2^n}$: bitwise conversion



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- $x = (4 \cdot 4 + 2 \cdot 0 + 0) + (4 \cdot 2 + 2 \cdot 0 + 0) + (4 \cdot 3 + 2 \cdot 0 + 1) = 16 + 8 + 12 + 1 = 37 = 5 \pmod{2^3}$
- Need n share conversions for n-bit numbers.

Sharing in $\mathbb{Z}_{2^n}$ is a sum of sharings over $\mathbb{Z}_2^n$

$$\mathbb{Z}_{2^3}\text{:}\ \boxed{1\,|\,0\,|\,1} + \boxed{0\,|\,1\,|\,1} = \boxed{0\,|\,0\,|\,0}$$

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$$\begin{array}{lc}
\mathbb{Z}_2^3: & \boxed{1}\boxed{0}\boxed{1} \oplus \boxed{0}\boxed{0}\boxed{0} = \boxed{1}\boxed{0}\boxed{1} \\
 & \boxed{0}\boxed{0}\boxed{0} \oplus \boxed{0}\boxed{1}\boxed{1} = \boxed{0}\boxed{1}\boxed{1} \\
\hline
 & \boxed{1}\boxed{1}\boxed{0} \oplus \boxed{1}\boxed{1}\boxed{1} = \boxed{1}\boxed{0}\boxed{1}
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This gives us conversion: $\mathbb{Z}_{2^n} \to \mathbb{Z}_2^n$

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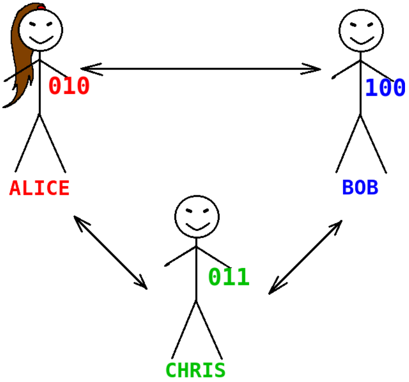
- We can apply the same idea for $\mathbb{Z}_2^n \to \mathbb{Z}_{2^n}$.

The second method for $\mathbb{Z}_2^n \to \mathbb{Z}_{2^n}$: bitwise subtraction



- $x = 010 \oplus 100 \oplus 011 = 101_2 = 5$

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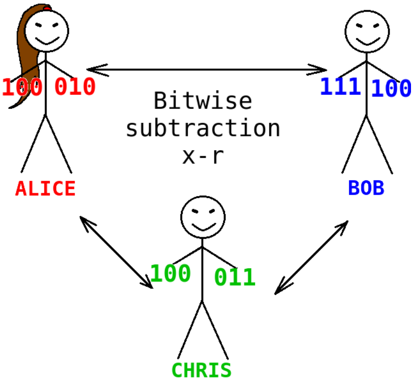
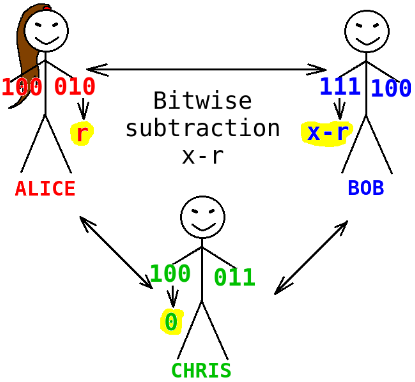


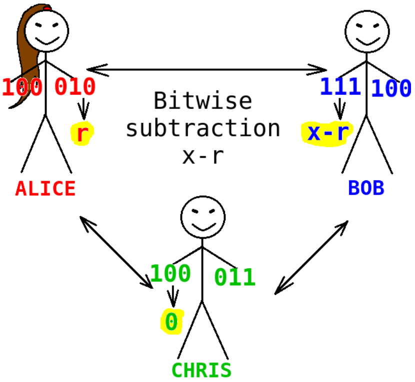
- $x = 010 \oplus 100 \oplus 011 = 101_2 = 5$
- $r = 100 \oplus 111 \oplus 100 = 111_2 = 7$

The second method for $\mathbb{Z}_2^n \to \mathbb{Z}_{2^n}$: bitwise subtraction



- $x = 010 \oplus 100 \oplus 011 = 101_2 = 5$
- $r = 100 \oplus 111 \oplus 100 = 111_2 = 7$
- Open r to Alice, and x - r to Bob, let Chris take 0.

The second method for $\mathbb{Z}_2^n \to \mathbb{Z}_{2^n}$: bitwise subtraction



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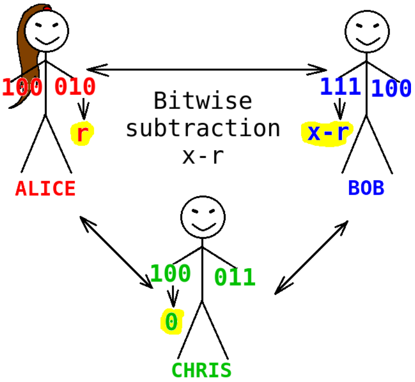

- $x = 010 \oplus 100 \oplus 011 = 101_2 = 5$
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- Open r to Alice, and x - r to Bob, let Chris take 0.
- r + x - r + 0 = x
- Need one bitwise subtraction of an *n*-bit number.

The bitwise subtraction protocol

- Let $[\![x]\!] = (x, x, x)$ denote a secret-shared value x.

Algorithm 1: Secure subtraction of XOR-shared numbers

Data: $n \in \mathbb{N}$, shared bits $[\![x_0]\!], \ldots, [\![x_{n-1}]\!], [\![y_0]\!], \ldots, [\![y_{n-1}]\!] \in \mathbb{Z}_2$

$[\![c_0]\!] = 0$
for $k = 0$ **to** $n - 1$ **do**
  $[\![s_k]\!] = [\![x_k]\!] \oplus [\![y_k]\!] \oplus [\![c_k]\!]$;
  $[\![c_{k+1}]\!] = (([\![c_k]\!] \oplus [\![y_k]\!]) \wedge ([\![c_k]\!] \oplus [\![s_k]\!])) \oplus [\![c_k]\!]$
end
Return $[\![s_0]\!], \ldots, [\![s_{n-1}]\!]$.

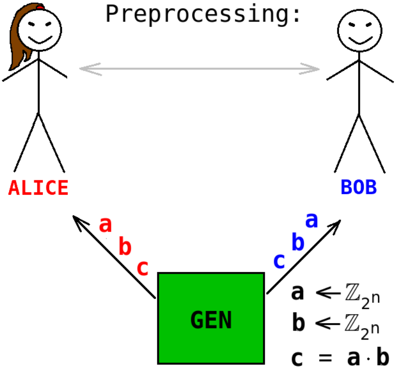
Uses n secure AND-s.

Comparing the two methods

- Bitwise subtraction: generic, uses only secure AND / XOR.
  - The complexity is n ANDs.
  - (+ opening of x and $x - r$).
- Bitwise conversion: elaborated, uses 3-party benefits.
  - The complexity is n conversions $\mathbb{Z}_2 \to \mathbb{Z}_{2^n}$.
  - Was preferred by Sharemind system.

Assisted 2-party computation using correlated randomness

- Beaver triples: $([\![a]\!], [\![b]\!], [\![c]\!])$ where $a, b \xleftarrow{\$} \mathbb{Z}_{2^n}$, $c = a \cdot b$.
  - Complexity of generating one n-bit triple is $O(n)$.



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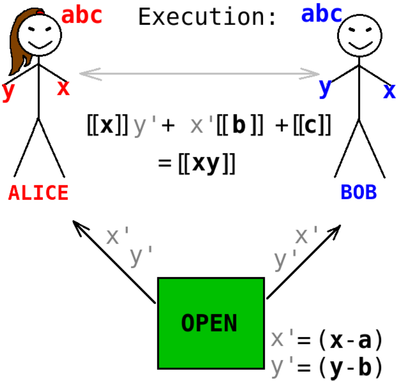
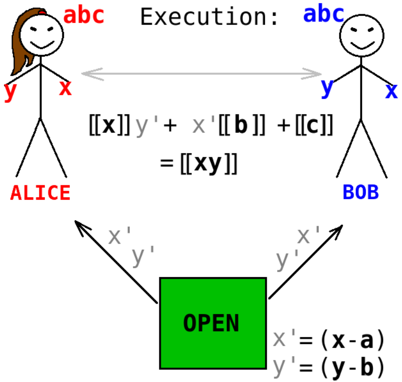
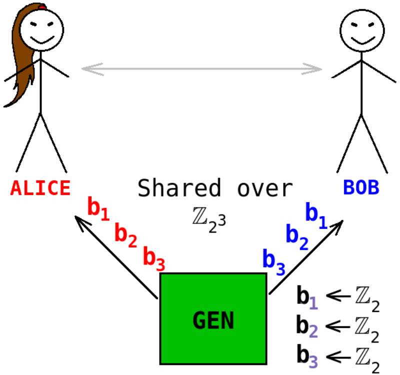


Assisted 2-party computation using correlated randomness

- Beaver triples: $([\![a]\!], [\![b]\!], [\![c]\!])$ where $a, b \xleftarrow{\$} \mathbb{Z}_{2^n}$, $c = a \cdot b$.
  - Complexity of generating one n-bit triple is $O(n)$.
- Trusted bits: $[\![b]\!]$ where $b \xleftarrow{\$} \mathbb{Z}_2$ is shared over $\mathbb{Z}_{2^n}$.
  - Complexity of generating one trusted bit is $O(n)$.



Using trusted bits for $\mathbb{Z}_2^n \to \mathbb{Z}_{2^n}$ (previous work)



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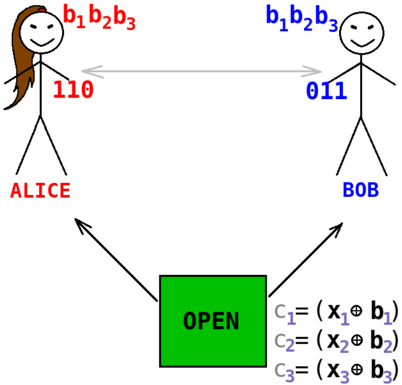

- $x = 110 \oplus 011 = 101_2$
- $x_1 = 1, x_2 = 0, x_3 = 1$

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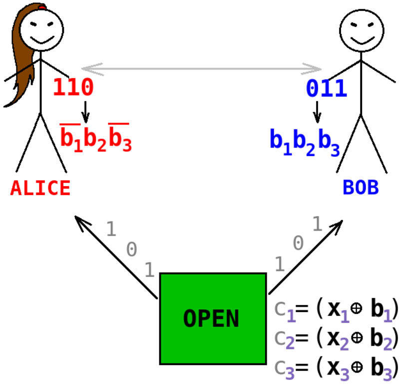
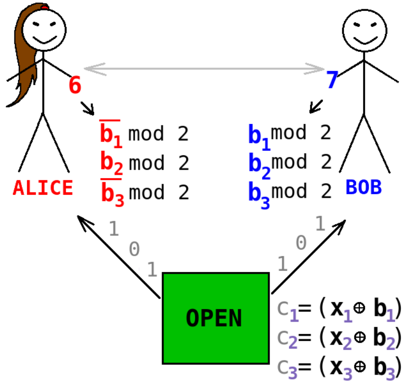


- $x = 110 \oplus 011 = 101_2$
- $x_1 = 1,\ x_2 = 0,\ x_3 = 1$
- $x = \overline{b_3} b_2 \overline{b_1} \oplus b_3 b_2 b_1 = 101_2$
- $x = (4 \cdot \overline{b_3} + 2 \cdot b_2 + \overline{b_1}) + (4 \cdot b_3 + 2 \cdot b_2 + b_1) = 5 \pmod{2^3}$

Using trusted bits for $\mathbb{Z}_{2^n} \to \mathbb{Z}_2^n$ (previous work)



- $x = 6 + 7 = 5 \pmod{2^3} = 101_2$
- $x_1 = 1,\ x_2 = 0,\ x_3 = 1$
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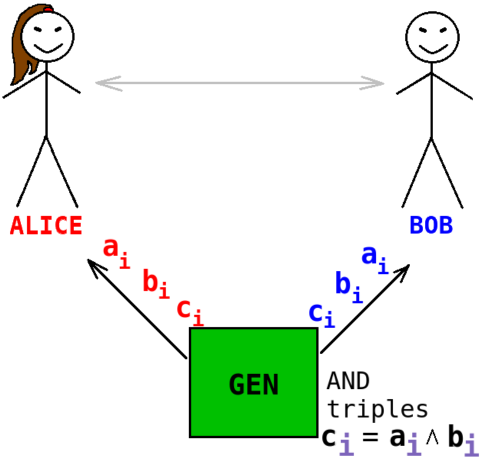
Using Beaver triples for $\mathbb{Z}_2^n \to \mathbb{Z}_{2^n}$ (this work)



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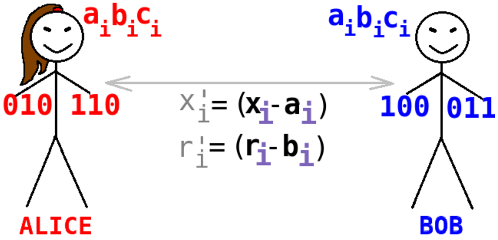

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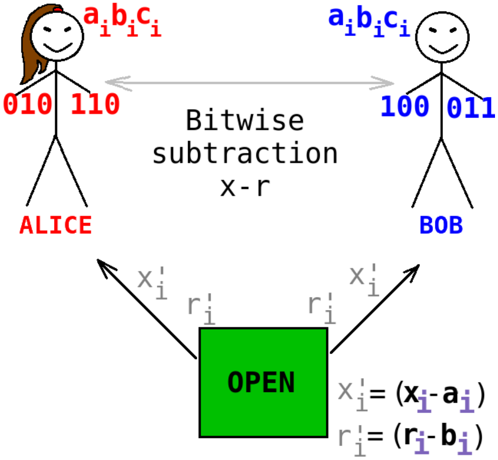

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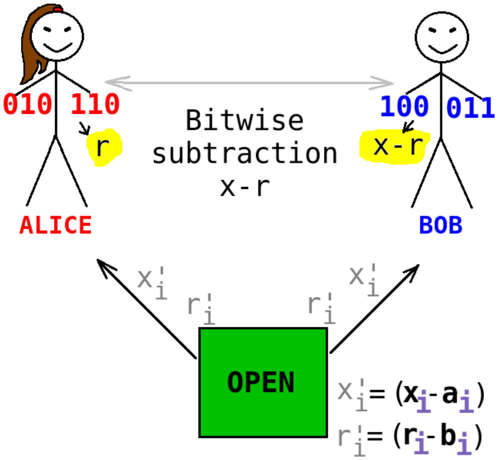

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Using Beaver triples for $\mathbb{Z}_{2^n} \to \mathbb{Z}_2^n$ (this work)

- $x = 6 + 7 = 13 = 5 \pmod{2^3}$

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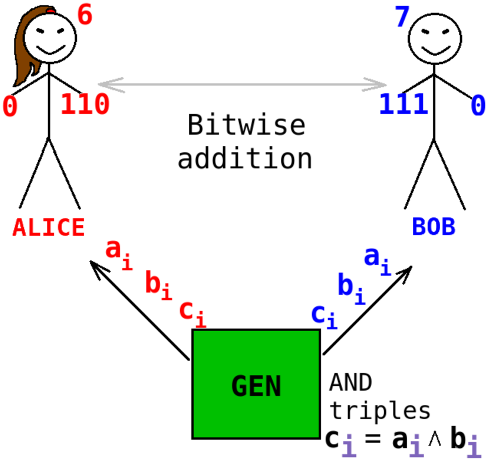

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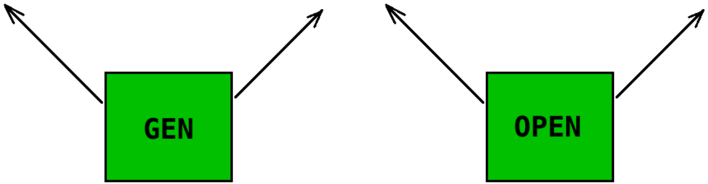
Summary for assisted 2-party $\mathbb{Z}_2^n \leftrightarrow \mathbb{Z}_{2^n}$ conversions

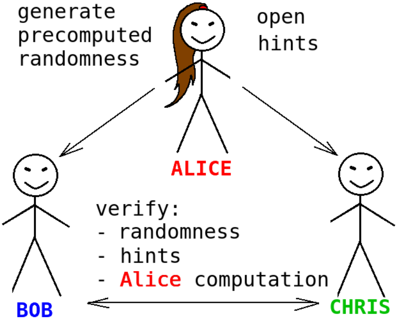
	correlated randomness	communication cost	
		preprocessing	verification
Bitwise conversion	Beaver triples	$2 \cdot n^2$	n
Bitwise addition	trusted bits	$2 \cdot 3n$	$2n$

(without taking into account the inside of the *Magic Box*).



Sharemind 3-party actively secure computation

1. Precompute sufficiently Beaver triples and trusted bits.
2. Run 3-party passively secure protocol.
3. Run assisted 2-party actively secure protocol to verify each party's computation.
   - The prover acts as an assistant of the verifiers.



Improvements for active security

Integer operation	Total bit communication for 32-bit integer protocols	
	Using bitwise conversion (old)	Using bitwise addition (new)
$[\![x]\!] \cdot [\![y]\!]$	192 : 768 : 4034 *1 : 4 : **21***	192 : 768 : 4034 *1 : 4 : **21***
$[\![x]\!] / [\![y]\!]$	31.7k : 275k : 28M *1 : 8 : **884***	31.7k : 358k : 1.7M *1 : 11 : **54***
$[\![x]\!] \ll [\![y]\!]$	1296 : 9374 : 64.6k *1 : 7 : **50***	1296 : 9758 : 50.9k *1 : 7 : **39***
$[\![x]\!] \gg [\![y]\!]$	2384 : 53k : 303.6k *1 : 22 : **127***	2608 : 53.0k : 278.8k *1.1 : 22.2 : **117***
$[\![x]\!] \gg y$	1092 : 9946 : 184.1k *1 : 9 : **169***	1092 : 12.5k : 61.2k *1 : 11 : **56***
$[\![x]\!] = [\![y]\!]$	218 : 872 : 14.3k *1 : 4 : **66***	218 : 1000 : 5252 *1 : 4 : **24***

Result is of the form x : y : z where

- x is the online passively secure phase;
- y is the verification phase;
- z is the preprocessing phase.