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PROBLEM OF FORBIDDEN SET[1]

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In [1; 2] are considered many problems about second and third-order rational difference equations. One of problems is to find the forbidden set. Usually are studied, for example, third-order rational difference equations

$$x_{n+1} = \frac{\alpha + \beta x_n + \gamma x_{n-1} + \delta x_{n-2}}{A + Bx_n + Cx_{n-1} + Dx_{n-2}}, \quad n = 0, 1, ... \tag{1}$$

with nonnegative parameters $\alpha, \beta, \gamma, \delta, A, B, C, D$ and with arbitrary nonnegative initial conditions x_{-2}, x_{-1}, x_0 such that the denominator is always positive. But what happens if we choose arbitrary initial conditions? Or some parameters are negative?

Definition 1. The set of initial conditions through which the denominator $A + Bx_n + Cx_{n-1} + Dx_{n-2}$ in Eq(1) will become zero for some value of $n \geq 0$ is called the forbidden set F of Eq(1).

For some rational difference equation the forbidden set has bee found. For example, Riccati difference equation $x_{n+1} = \dfrac{\alpha + \beta x_n}{A + Bx_n}$ has been widely investigated.

We consider Pielou's equation

$$x_{n+1} = \frac{\beta x_n}{A + Cx_{n-1}}.$$

Our aim is to find the forbidden set of Pielou's equation and thereby to solve Open Problem 5.24.2 given in [1].

REFERENCES

[1] E. Camouzis, G. Ladas. *Dynamics of Third-Order Rational Difference Equations with Open Problems and Conjectures.* Chapman&Hall/CRC, 2008.

[2] M.R.S. Kulenovic, G. Ladas. *Dynamics of Second Order Rational Difference Equations. With Open Problems and Conjectures.* Chapman&Hall/CRC, 2002.

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LINEARIZATION OF THE NONLINEAR DIFFERENCE EQUATIONS

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We consider the nonlinear difference equation in the form

$$x_{n+l} = f(x_{n+l-1}, \dots, x_{n-k}), \quad n = 0, 1, \dots \quad k \in \{0, 1, 2, ...\}, \quad l \in \{1, 2, ...\}, \tag{1}$$

and the initial conditions are real numbers. The linearization of this equation can be written in the following form

$$x_{n+l} = \sum_{i=1-l}^{k} g_i x_{n-i}, \quad n = 0, 1, \dots, \tag{2}$$

where $g_i : \mathbf{R}^{k+l} \to \mathbf{R}$.

Such identities are often used to study the behaviour of the solutions of the difference equation and building blocks of semicycle analysis, method of invariants and rate of convergence, see [1], [2], [3], [4].

The author study the global stability character, the periodic nature, and boundedness of solutions of a nonlinear difference equation depending on a linearization equality. In this talk we consider some types of the nonlinear difference equation and illustrate various examples.

REFERENCES

[1] E. J. Janowski, M. R. S. Kulenović. Attractivity and Global Stability for Linearizable Difference Equations. *Computers & Mathematics with Applications*, **57** (9), 2009, pp.1592-1607.

[2] E. J. Janowski, M. R. S. Kulenović, E. Silić. Periodic Solutions of Linearizable Difference Equations. *International Journal of Difference Equations*, **6** (2), 2011, pp.113-125.

[3] M.R.S. Kulenović, M.Mehuljić. Global Behavior of Some Rational Second Order Difference Equations. *International Journal of Difference Equations*, **7** (2), 2012, pp.153–162.

[4] E. J. Janowski, M. R. S. Kulenović. Existence of a Period - Two Solution in Linearizable Difference Equations. *International Journal of Difference Equations*, Discrete Dynamics in Nature and Society, 2013.

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ON SMOOTHING SPLINES WITH SIGN CONSTRAINTS

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The talk deals with the following conditional minimization problem

$$||Tx||^2 + ||R(Ax - v)||^2 \longrightarrow \min_{l_i x \geq 0,\ i=1,\ldots,m},$$

where $T:\ X\ \to\ Y,\ A : X \to I\!R^n$ and $B = (l_1, \ldots, l_m) : X \to I\!R^m$ are linear continuous operators in Hilbert spaces X and Y, $R = diag(\sqrt{\rho_i})_{i=1,\ldots,n}$ is the diagonal matrix with positive weights and $\boldsymbol{v} \in I\!R^n$. This problem is a special case of smoothing problems in convex sets.

We investigate the existence and characterization of its solutions. More detailed results are obtained for some specific problems. In particular, we consider approximation in Sobolev space and discuss the possibility to apply this problem to shape preserving smoothing under constraints on positivity and/or monotonicity of solutions.

This study is closely related to our previous works [1], [2].

REFERENCES

[1] S. Asmuss and N. Budkina. Splines in convex sets under constraints of two-sided inequality type in a hiperplane. *Mathematical Modelling and Analysis*, **13** (4):461–470, 2008.

[2] S. Asmuss and N. Budkina. On smoothing problems with one additional equality condition. *Mathematical Modelling and Analysis*, **14** (2):159–168, 2009.

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BIFURCATION OF LIMIT CYCLES IN HAMILTONIAN SYSTEMS

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We consider the Hamiltonian system

$$\begin{cases} x' &= y, \\ y' &= -g(x), \end{cases} \tag{1}$$

where $g(x) = x^3(x^2 - 1)$ [2].

It was shown in [2] how to perturb system (1) in order the perturbed system to have two limit cycles (exactly one limit cycle in each of two period annuli).

We define the perturbed system

$$\begin{cases} x' &= \varepsilon a(x,y)g(x) + y, \\ y' &= \varepsilon a(x,y)y - g(x), \end{cases} \tag{2}$$

where ε is a small parameter and $a(x, y)$ is a specially chosen coefficient. The perturbed system (2) has more than one limit cycle in each of two period annuli.

REFERENCES

[1] S. Atslega and F. Sadyrbaev. On periodic solutions of Liénard type equations. *Mathematical Modelling and Analysis*, **18** (2011), 708-716.

[2] R. Ashegi, H.R.Z. Zangeneh. Bifurcations of limit cycles for a quintic Hamiltonian system with a double cuspidal loop. *Computers and Mathematics with Applications*, **59** (2010), 1409-1418.

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TRICHOTOMIES OF RATIONAL DIFFERENCE EQUATIONS

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In [3] Palladino investigated periodic trichotomy results for the k^{th} order rational difference equations in the form

$$x_n = \frac{\alpha + \sum_{i=1}^{k} \beta_i x_{n-i}}{A + \sum_{i=1}^{k} B_i x_{n-i}}, n = 1, 2, \ldots \tag{1}$$

with non-negative parameters and non-negative initial conditions. The study is based on the connection between parameter A and $\sum_{i=1}^{k} \beta_i x_{n-i}$ of equation (1) as well as on roots of the characteristic equation. Periodic trichotomies also were studied in [1], [2].

Example (see [1]). Consider the difference equation

$$x_{n+1} = \frac{x_{n-2}}{A + Bx_n + Cx_{n-1}}, n = 1, 2, \ldots \tag{2}$$

with non-negative parameters and $B + C > 0$. Then the solutions of equation (2) have the following period-three trichotomy behavior:

- When $A > 1$ every solution of equation (2) converges to zero.
- When $A = 1$ every solution of equation (2) converges to a periodic solution of period 3.
- When $0 \leq A < 1$, then for some initial conditions equation (2) has unbounded solutions.

In this talk we present some rational difference equations that posses similar properties accordingly to the values of the roots of characteristic equation.

REFERENCES

[1] E.Camouzis, G.Ladas. Dynamics of Third-Order Rational Difference Equations with Open Problems and Conjectures. *Chapman & Hall, CRC*, 2007.

[2] Dz.Burgic, S.Kalabusic, M.R.S.Kulenovic. Period-Two Trichotomies of a Difference Equation of Order Higher than Two. *Sarajevo Journal of Mathematics*, **4** (16), 73–90, 2008.

[3] F.J.Palladino. On Periodic Trichotomies. *Difference Equations and Applications*, **15** (6), 605–620, 2009.

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ARITHMETICAL SUBSEQUENCES OF FINITELY GENERATED BI-IDEALS AND BOUNDED BI-IDEALS[1]

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The classical Van der Waerden theorem [1] states that for each right infinite word $x = x_0x_1 \cdots x_n \cdots$ over a finite alphabet Σ there exist arbitrarily long arithmetical progressions $l, l+p, \ldots, l+np$ such that the letters in the corresponding positions are all equal ($x_l = x_{l+p} = \cdots = x_{l+np}$). An obvious generalization is to look at infinite sequences of letters in positions corresponding to some arithmetical progression — so called arithmetical subsequences. While there are words of which all arithmetical subsequences are aperiodic — such as the Thue-Morse word (see, [2]) and the Sturmian words (see [3] for a survey on Sturmian words), — there are also words that contain periodic arithmetical subsequences. For example, Toeplitz words [4] obviously contain periodic arithmetical subsequences.

In this talk we consider the aperiodicity of all arithmetical subsequences of finitely generated bi-ideals and bounded bi-ideals (see, e.g., [5], [6], [7] for more on bi-ideals).

REFERENCES

[1] B. L. van der Waerden, Beweis einer Baudetschen Vermutung, *Nieuw Arch. Wiskd.* **15** (1927), 212–216.

[2] A. O. Gelfond, Sur les nombres qui ont des propriétés additives et multiplicatives données, *Acta Arith.* **13** (1968), 259-265.

[3] M. Lothaire, *Algebraic Combinatorics on Words*, Encyclopedia Math. Appl. 90, Cambridge University Press, 2002.

[4] J. Cassaigne and J.Karhumäki, Toeplitz words, generalized periodicity and periodically iterated morphisms, In *Computing and Combinatorics: Proc COCOON 1995*, Lecture Notes in Comput. Sci., Vol. 959, Springer, 1995, pp. 244–253.

[5] J. Buls and A. Lorencs, From bi-ideals to periodicity, *RAIRO Theor. Inform. Appl.* **42** (2008), 467–475.

[6] E. Cers, *Finitely Generated Bi-ideals and the Semilattice of Machine Invariant ω-words*, PhD Thesis, University of Latvia, 2012.

[7] A. Lorencs, The identity problem of finitely generated bi-ideals, *Acta Inform.* **49** (2012), 105–115.

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SYMBOLIC AGGREGATE APPROXIMATION (SAX) METHOD FOR TIME SERIES REPRESENTATION[1]

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A symbolic representation of time series have developed in last decade, because amount of the data very often is extremely high. Somehow it is necessary to reduce the dimensionality of the time series for an easier way to explore or process data. A piecewise aggregate approximation (PAA), which is described in [1], allow to reduce the dimension n of time series by dividing the given data into w, where $w < n$ (typically $w << n$ and $m = \frac{n}{w}$), equal sized frames. From a given time series $C = c_1, \dots, c_n$, we get PAA:

$$\overline{c_i} = \tfrac{1}{m} \sum_{j=m(i-1)+1}^{mi} c_j$$

If we introduce an alphabet $A = \{a_1, a_2, \dots, a_s\}$, then the breakpoints $B = \beta_1, \dots, \beta_{s-1}$ is defined by relating them to the area under normalized $N(0,1)$ Gaussian curve. Now we are able to get a symbolic aggregate aproximation (SAX) $\hat{C} = \hat{c_1}, \dots, \hat{c_w}$ for the time series from PAA:

$$\hat{c_i} = a_j, \text{ iff } \beta_{j-1} \leq \overline{c_i} < \beta_j.$$

The minimum distance between SAX of two time series $\hat{Q}$ and $\hat{C}$ can be found as:

$$MINDIST(\hat{Q}, \hat{C}) = \sqrt{m}\sqrt{\sum_{i=1}^{w} dist(\hat{q}_i, \hat{c}_i)^2}.$$

In this talk we compare and try to analyze this distance function of SAX and the distance function of two words x and y from the combinatorics on words point of view [2], which is:

$$d(x,y) = \begin{cases} 0, & \text{if } x = y, \\ 2^{-n}, & \text{otherwise,} \end{cases}$$

where $n = min\{i : x[i] \neq y[i]\}$.

REFERENCES

[1] J. Lin, E. Keogh, S. Lonardi and B. Chiu. A symbolic representation of time series with implications for streaming algorithms. *In: Proceedings of the 8th ACM SIGMOD workshop on Research issues in data mining and knowledge discovery*, 2003., p.2-11.

[2] J.P. Allouche and J. Shallit. . *Automatic Sequences*, Cambridge University Press, 2003.

[1]This work was supported by ESF project 2013/0024/1DP/1.1.1.2.0/13/APIA/VIAA/045

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TRANSIENT HEAT TRANSFER IN SYSTEM WITH DOUBLE WALL AND DOUBLE FINS

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In this talk I consider transient heat conduction in a system with double wall and double fins. The problem is examined when assigning third type linear boundary conditions and boiling conditions on the fins' surface. Conservative averaging and finite difference methods are applied to the given problem to construct a numerical solution. Finally, the numerical results for both stationary (see [1], [2]) and transient cases are compared.

REFERENCES

[1] T. Bobinska, M. Buike and A. Buikis. Heat Transfer with Partial Boiling in System with Double Wall and Double Fins. In: *Proceedings of the 11th WSEAS Int. Conf. on Heat Transfer, Thermal Engineering and Environment, Vouliagmeni, Greece, 2013*, WSEAS Press, 2013, 132 – 137.

[2] T. Bobinska, M. Buike, A. Buikis and H.H. Cho. Stationary Heat Transfer in System with Double Wall and Double Fins. In: *Proceedings of the 11th WSEAS Int. Conf. on Data Networks, Communications, Computers and the 5th WSEAS Int. Conf. on Materials Science, Sliema, Malta, 2012*, WSEAS Press, 2012, 260 – 265.

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ON P - CLOSED SPACES[1]

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A notion of p -closedness defined analogically to the notion of C - closedness in [2] is studied.

Let $p \in \omega^*$ where $\omega^* = \beta\omega \setminus \omega$ is Stone-$\hat{C}$ech growth of a countable discrete space ω. We consider p as a free ultrafilter on the set of all positive integers ω.

Definition 1 [1] Let $\{x_n : n \in \omega\}$ be a sequence in topological space (X, T). An element x is said to be a p-limit point of the sequence if $\{n : x_n \in V_x\} \in p$ for any neighborhood V_x of x . In this case one can say that $\{x_n : n \in \omega\}$ p-converges to x and we write $x = p\text{-}\lim x_n$.

Definition 2 [1] Topological space (X, T) is called p-compact space if any sequence $\{x_n : n \in \omega\}$ p -converges to some point x.

Definition 3 [3] A topological space is called p-sequential if for any its nonclosed subset A there is a point $x \in \bar{A} \setminus A$ and a sequence $\{x_n : n \in \omega\} \subset A$ such that $x = p\text{-}\lim x_n$. A subspace of a p-sequential space is called p-subsequential.

Definition 4 A topological space is called p-closed provided each its p-compact subset is closed.

Theorem 1 Every Hausdorff p-sequential space is p-closed. Specifically, every sequential space is p-closed.

Theorem 2 A Hausdorff space is a p-closed space whenever each its p-compact subset is p-sequential space.

Theorem 3 A quitient of a p-subsequential space is p-subsequential.

Theorem 4 Let (X, T) be topological space of countable tightness such that each of its countable subspaces is a p-subsequential space. Then (X, T) is a p-subsequential space.

REFERENCES

[1] VA.R.Bernstein. A new kind of compactness for topological spaces, *Fund. Math.* 166 185-193, 1970

[2] M. Ismail., P. Nyikos. On spaces in which countably compact sets are closed, and hereditary properties, *Topology Appl.* 11 281-292, 1980

[3] A.P. Kombarov, O kompaktnosti i sekvencialjnosti po mnozhestvu uljtrafiljtrov. *Vestnik Moskovskogo Univ., Ser. Mat. Meh.* 5 15-18, 1985 (Russian)

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HAMILTON SYSTEM IN BIOLOGICAL POPULATIONS

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Two dimensional differential system is considered which includes population dynamics system of predator-prey, symbiosis and competitive type. We provide condition for dissipativity [1].

Also we consider Hamiltonian system of the same structure (1) which is not dissipative

$$\begin{cases} \dot{x} &=& x(-ax - by + c), \\ \dot{y} &=& y(\quad ax + by - c) \end{cases} \tag{1}$$

where $a, b, c \neq 0$. Visualizations and graphics are provided.

REFERENCES

[1] Shair Ahmad and Alan C. Lazer. On a property of a generalized Kolmogorov population model. *Discrete and continuous dynamical systems*, **vol. 33 Number 1** pp. 1-6, 2013.

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RIEMANNIAN GEOMETRY OF SHAPE SPACES

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According to Kendall [1], shape is that, which "is left when the effects associated with translation, scaling and rotation are filtered away". The simplest example of a shape space is the set of triangles modulo similitudes. The set of unparametrized plane curves is another example, an infinite-dimensional one.

This talk will explore various shape spaces [2], concentrating mainly on the space of unparametrized curves. Riemannian metrics are usually defined via Riemannian submersions from a larger, computationally more tractable preshape space. In the case of unparametrized curves this larger space is the space of parametrized curves.

A particular family of metrics is the class of Sobolev-type metrics, which are metrics of the form

$$G_c(h,k) = \int_{S^1} \sum_{j=0}^{n} a_j \langle D_s^j h, D_s^j k \rangle \, \mathrm{d}s \,.$$

Here c is a curve, h, k are tangent vectors, that is $\mathbb{R}^2$-valued vector fields along c, $D_s = \partial_\theta / |c'|$ is the arc-length derivative and by $\mathrm{d}s = |c'| \, \mathrm{d}\theta$ we denote arc-length integration. Sobolev-type metrics arose from the need of strengthen the L^2-metric, which was found to have vanishing geodesic distance. We will discuss the mathematical properties of Sobolev metrics with particular emphasis on completeness results [3].

REFERENCES

[1] D. G. Kendall. Shape manifolds, Procrustean metrics, and complex projective spaces. *Bull. London Math. Soc.*, **16(2)** 81–121, 2984.

[2] M. Bauer, M. Bruveris, P. W. Michor. Overview of the geometries of shape spaces and diffeomorphism groups. *J. Math. Imaging Vis.*, 2014.

[3] M. Bruveris, P. W. Michor, D. Mumford. Geodesic completeness for Sobolev metrics on the space of immersed plane curves. arXiv:1312.4995, 2013.

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NEURONS MODELLING[1]

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In recent years, cellular neural networks have been extensively studied and found many important applications in different areas, such as psychophysics, perception, robotics, adaptive pattern recognition, image processing and associate memory. We have investigated one neuron model with different signal functions. Origin of the model comes from [3] but idea of finding periodic orbits of the model first was demonstrated in [4]. We have investigated a difference equation

$$x_{n+1} = \beta x_n - g(x_n). \tag{1}$$

By [3] x denotes the activation level of a neuron, β is interpreted as an internal decay rate and g is a signal function. According to parameter β we obtain different behaviour of solutions of difference equation (1). Signal function play an important role. In our work we used step functions with two and three thresholds (see [1]) therefore in fact we investigate one dimensional discontinuous piecewise linear map. If the parameter $\beta > 1$ we have observed chaotic orbits. Chaotic orbit means not predictable changes of activation level.

Knowing that a single neuron model does not exist in reality, we note an interesting fact that similar functions as in our model are used to study the power spectral density of signals with applications in telecommunications and transmission security ([2]).

REFERENCES

[1] A. Anisimova, M. Avotina and I. Bula. Periodic Orbits of Single Neuron Models with Internal Decay Rate $0 < \beta \leq 1$. *Mathematical Modelling and Analysis*, **18,** 325–345, 2013.

[2] K. Feltekh, Z.B. Jemaa, D. Fournier-Prunaret and S. Belghith. Border collision bifurcations and power spectral density of chaotic signals generated by one-dimensional discontinuous piecewise linear maps. *Communications in Nonlinear Science and Numerical Simulation*, **19,** 2771–2784, 2014. dx.doi.org/10.1016/j.cnsns.2013.12.029

[3] J. Wu. *Introduction to neural Dynamics and Signal Transmission Delay.* De Gruyter, Berlin, 2001.

[4] Z. Zhou. Periodic Orbits on Discrete Dynamical Systems. *Computers and Mathematics with Applications*, **45,** 1155-1161, 2003. dx.doi.org/10.1016/S0898-1221(03)00075-0

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AUTOMATON SEMIGROUPS AND GROUPS

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Automaton (semi)groups — short for semigroups generated by Mealy machines and groups generated by invertable Mealy machines — were formally introduced half a century ago (for details, see [1]).

Let $V = \langle Q, A, A, \circ, * \rangle$ be machine and

$$\dot{q} : A \to A : a \mapsto q * a$$
$$\ddot{a} : Q \to Q : q \mapsto q \circ a$$

A machine V is called invertible if all functions $\dot{q}$ are permutations of A and reversable if all functions $\ddot{a}$ are permutations of Q. For $w = q_1 q_2 \ldots q_n \in Q^n$ set $\dot{w} = \dot{q}_1 \dot{q}_2 \ldots \dot{q}_n$. The semigroup of mappings from $A^* \to A^*$ generated by $\dot{q}, q \in Q$, is called the semigroup generated by V. When V is invertible it is called the group generated by V.

Is the finiteness problem for automaton groups undecidable? Recently Pierre Gilbert [2] had announced that the finiteness problem for automaton semigroups is undecidable.

REFERENCES

[1] R.I. Grigorchuk, V.V. Nekrashevich, and V.I. Sushchanskii. Automata, dynamical systems and groups. *Tr. Mat. Inst. Steklova*, **231** 134 – 214, 2000.

[2] Pierre Gillibert . The finiteness problem for automaton semigroups is undecidable. In: *http://arxiv.org/pdf/1304.2295.pdf*, 2013 , 7 pages .

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MEAN SQUARE LYAPUNOV EXPONENT FOR MARKOV MULTIPLICATIVE COCYCLES

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For the first time the concept of Lyapunov exponent for stochastic dynamical systems has been proposed in [1]. The author has proved that any solution $x(t,x_0)$ of differential equation in $\mathbb{R}^d$

$$\frac{dx}{dt} = A(\xi(t))x, x(0) = x_0 \neq 0 \tag{1}$$

where $\{\xi(t)\}$ is ergodic stationary process satisfying condition $\mathbf{E}\{||A(\xi(0)||\} < \infty$, has exponential non-random growth rate (Lapunov exponent) $\lambda(x_0) := \lim\limits_{t\to\infty} \frac{1}{t} ln|x(t,x_0)|$, which can take only finitely many values $\lambda_1 < \lambda_2 < \cdots < \lambda_m$ (*Lyapunov spectrum*). Later this approach has been discussed by many authors (see, for example, the Conferences [2] and [3]). It has been noted that for asymptotic analysis of stochastic dynamical systems more efficiently is the concept of p-moment Lyapunov exponent given by formula $\lambda^{(p)}(x_0) := \lim\limits_{t\to\infty} \frac{1}{t} ln\mathbf{E}\{|x(t,x_0)|^p\}, p > 0$. Our paper deals with second moment Lyapunov exponents for linear multiplicative cocycles defined as two parametric semigroup of Cauchy matrix family for linear stochastic Ito equations, impulse type stochastic equations, and linear differential equation (1) with coefficients continuously dependent on ergodic Markov processes $\{\xi(t)\}$. We have proved that the conditional covariance matrices

$$q(t,y) := \mathbf{E}\{x(t)x(t)^T/\xi(t) = y\}, \quad t \geq 0, y \in \mathbb{Y}$$

satisfy differential equation

$$\frac{\partial}{\partial t} q(t,y) = (\mathbf{A}q)(t,y)$$

in the space of continuous symmetric matrix-functions. Taking an advantage of covariance matrix-functions positivity we may look for second moment Lyapunov spectrum $\{\lambda_k^{(2)}\}$ as the real eigenvalues of operator $\mathbf{A}$. This approach permits us to propose convenient for application algorithm of mean square stability analysis.

REFERENCES

[1] V.I.OseledecA multiplicative ergodic theorem. Lyapunov characteristic numbers for dynamical systems. *Trans. Moscow Math. Soc., 19 (1968)*197-231

[2] L. Arnold and V. Wihstutz(Eds.) *Lyapunov Exponents.Proceedings of a Conference held Bremen, November 12-15, 1984* Springer-Verlag, Berlin Heidelberg. 1986

[3] L. Arnold, H. Crauel, J. Eckmann (Eds.): *Lyapunov Exponents.Proceedings of a Conference held in Oberwolfach, May 28 - June 2, 1990* Springer-Verlag, Berlin Heidelberg. 1991.

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BOUNDARY VALUE PROBLEMS IN MATHEMATICAL MODELLING OF PRE-STEADY STATE ENZYME-CATALYSED REACTIONS [1]

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Mathematical modelling of fixed bed reactors using immobilized enzymes is important for a wide range of applications. The reaction rate R_p involved in the governing equations often has a non-Michaelis-Menten form which implies non uniqueness of boundary value problems (for example, see [1]). Introducing pre-steady state relaxation time τ_r and using the modification of Fick's law for the mass flux J in the mass balance equation (see, also [1]) is possible to obtain the problems for partial differential equation of hyperbolic type. Qualitative analysis of obtained problems is considered.

REFERENCES

[1] J. Cepitis. 5th Nordic-Baltic Agrometrics Conference Otepaa, Estonia, June 15–17,2005. In: *Mathematical Modelling of Biochemical Reactions for Certain Agriculture Production Processing*, U.Olsson and J.Sikk [Eds.], Swedish University of Agriculture Sciences, Uppsala, 2006, 35 – 40.

[1] This work was partially supported by the grant 345/2012 of the Latvian Council of Science

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SOLVED AND UNSOLVED PROBLEMS ON TETRADS

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Tetrad is a notion studied in combinatorial geometry. This term for the first time most likely appeared in [1]. One can find next contribution in famous Gardner's book [2]. Let us recall that a *tetrad* is a plane figure made of four congruent shapes, joined so that each one shares a boundary (of a positive measure) with each. It is worth noting that tetrads have some relevance with the four colour problem thus explaining appearance at the same time when legendary theorem was proved. In recent years tetrads have regained popularity mainly due to A. Cibulis, G. Sicherman and author. Webpage [3] is a very nice regularly updated account about tetrads with different properties. Additional information and some entertaining problems one can find in [4].

In the master thesis of the author special computer programmes have been elaborated to generate all n-omino tetrads for given n. Later these programmes were modified to handle other polyforms. In [5] the following theorem is proved: For each $n \geq 11$ there is an n-omino that forms a tetrad without holes. In this talk analogous theorem for polykites will be demonstrated. The general case for polyiamonds still remains unsolved. Non-trivial result, that for each polyomino P without holes a tetrad containing P as a unique hole there exists, will be discussed.

In [2] Gardner asks (a bit mathematically unclear though) whether full tetrad made of 5-gon exists. Can you prove or disprove this fact?

REFERENCES

[1] M. R. W. Buckley, Journal of Recreational Mathematics, 1975, 8.

[2] M. Gardner, Penrose Tiles to Trapdoor Ciphers, Freeman, 1989.

[3] http://userpages.monmouth.com/~colonel/tetrads/tetrads.html

[4] A. Cibulis, J. Čerņenoks, Tetrads it is not o easy, MCG Newsletter 3, October, 2012, pp. 23 - 27.[http://www.igmcg.org/files/newsletter_3.pdf]

[5] J. Čerņenoks, A. Cibulis, Application of IT in Mathematical Proofs and in Checking of Results of Pupils Research, Proceedings of the International Conference AICT 2013, pp. 172 - 177, Jelgava, Latvia. [http://aict.itf.llu.lv/proceedings/2013]

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SOME PROBLEMS OF EXTREMA AND POSSIBILITIES OF THEIR SOLVING BY ELEMENTARY METHODS

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There is a problem. Seek its solution.
We can find it by pure reason for in
Mathematics there is no ignorabimus.
David Hilbert

The attention is paid to Huygens' (Christian Huygens, 1629-1695) problem on collision of perfectly elastic balls, see e. g. [1], [2], [3]. If a ball $B(M,V)$ of mass M moving with velocity V strikes a motionless ball B_0 of mass m, the latter gives a speed $\nu = 2mV/(m+M)$. It turns out that by placing an intermediate ball of appropriate mass between these two balls it is possible to increase a speed ν of the ball B_0 after central collisions. What mass of the intermediate ball should be taken to obtain the maximum velocity of the ball B_0? And what about a general situation: how the masses m_j, $0 < m \le m_1 \le m_2 \le \ldots \le m_n \le M$ should be chosen so that the ball B_0 will acquire the maximum velocity after successive central collisions? On the basis of the laws of conservation of energy and momentum of a closed mechanical system one can obtain the following mathematical model.

$$\max \frac{m_1 \cdot \ldots \cdot m_n}{(m+m_1)(m_1+m_2)\ldots(m_n+M)} \cdot 2^{n+1}V = ?, m_1, \ldots, m_n \in [m, M].$$

By methods of differential calculus this problem has been solved in [2]. The problem for $n = 3$, $m = 1$, and $M = 16$ was offered to the 23^{rd} British Mathematical Olympiad, 1987, see, [4].

The next problem to be discussed is as follows: Cut n rectangles from the circle part in the first quadrant so that their edges are parallel to the coordinate axes and the sum of areas are maximal.

REFERENCES

[1] Cibulis A., Ekstrēmu uzdevumi, 1. d., Rīga, LU, 2007.

[2] Zorich V. A., Mathematical Analysis I, Springer-Verlag, 2004.

[3] `http://www3.nd.edu/~kbrading/Classes/Phil%2043713/Huygens%20on%20collisions.pdf`

[4] Gardiner A., The Mathematical Olympiad Handbook: An Introduction to Problem Solving, Oxford University Press, 1997.

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CHANGE-POINT ANALYSIS WITH APLICATIONS TO GENETICS

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Change-point detection is the problem of discovering time points at which properties of time-series data change. This covers a wide range of real-world problems in medicine, finance, genetics and has been actively discussed in the community of statistics (see [1]).

Usually statistical inference about change points has two aspects. The first is to detect if there is any change in the sequence of observed random variables. The second is to estimate the number of changes and their corresponding locations.

Our aim is to compare different methods of detecting change-points and to distinguish outliers from change-points.

REFERENCES

[1] J. Chen, A. K. Gupta. *Parametric statistical shange-point analysis.* Springer, New York, 2010.

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GENERALIZED PIVOTAL QUANTITIES AND GENERALIZED CONFIDENCE INTERVALS

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Weerahandi [1] introduced the idea of Generalized Pivotal Quantities and Generalized Confidence Intervals and derived confidence interval procedures for many problems where exact nontrivial frequentist intervals are unavailable. During the past few years generalized confidence intervals have been used by many authors to obtain useful inference procedures in nonstandard problems. Although Weerahandi [1] provided a number of examples illustrating the application of GPQs, he did not provide a systematic approach for finding them.

During the past few years, the idea of GCIs and generalized tests have been used by many authors to obtain useful inference procedures in nonstandard problems, but the problem of finding an appropriate pivotal quantity is a nontrivial task (see [2]). General method for constructing GPQs is as yet unavailable in the literature and each particular problem appears to require some ingenuity in constructing an appropriate GPQ or a test variable.

Our aim is to study GPQs and GCIs for ratio of two regression coefficients and construct them for Pamuk and Kunst-Mackenbach relative index of inequality (see [3]), which is a frequently used summary measure of socioeconomic inequality in health:

$$RII_P = \frac{-\beta}{\overline{y}}, \quad RII_{KM} = \frac{\alpha}{\alpha + \beta}$$

and the regression has the form

$$y = \alpha + \beta x.$$

For that purpose we will use the results of some related regression problems (see [4]).

REFERENCES

[1] S. Weerahandi. *Exact statistical methods for data analysis.* Springer, New York, 1995.

[2] H. K. Iyer, P. L. Patterson. A Recipe for Constructing Generalized Pivotal Quantities and Generalized Confidence Intervals *Technical report 2002/10.* Department of Statistics, Colorado State University, 2002.

[3] L. J. Hayes, G. Berry. Sampling variability of the Kunst-Mackenbach relative index of inequality. *Journal Epidemiology and Community Health*, **56** (10):762-765, 2002.

[4] I. Bebu, F. Seillier-Moiseiwitsch, T. Mathew. Generalized Confidence Intervals for Ratios of Regression Coefficients with Applications to Bioassays. *Biometrical Journal*, **51** (6):1047-1058, 2009.

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GRAPH STRUCTURE OF COMMUTING FUNCTIONS

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Composition of functions is an important binary operation in function sets. This operation is so omnipresent and important in mathematics that its basic property - associativity has been abstracted and accepted as a basic feature of algebraic structures such as groups. Functions commuting with respect to the composition operation have been studied for both purely theoretical and applied reasons. See [2] for an example of studies of commuting rational functions dating back to the early 20th century. Commutativity is the second most useful property of algebraic structures, its importance originates from commutativity of set-theoretic union and intersection. Commutativity of linear algebraic objects such as matrices with respect to multiplication has been studied since Frobenius. Generalizations of commuting functions, e.g. commuting matrices and operators, are important in applications such as quantum physics. Recently problems involving commuting permutations have appeared in algebra olympiads for university students, see Problem 1 in [1]. In this talk we describe graph structure of commuting functions and the results involve graph models of functions - functional graphs. The answer is well known for both functions being bijective in finite sets. The general case does not seem to have been described in the literature therefore some further study and description of commuting functions seems appropriate. These studies may provide additional links between algebra and discrete mathematics.

We describe endofunctions $g \in \mathcal{F}un(S)$ commuting with a given endofunction $f \in \mathcal{F}un(S)$. In terms of functional graphs we describe possible graph homomorphisms of functional graphs. Descriptions are given as correspondences $g \leftrightarrow (A, B, C, ...)$ where $A, B, ...$ are mappings or substructures related to S which are relatively easy to describe. Several subcases for finite sets S and corresponding generalizations for infinite S are discussed.

REFERENCES

[1] MSU olympiad. Algebra olympiad problems 2010. *http://halgebra.math.msu.su/Olympiad/2010/problems-10.pdf*, (in Russian).

[2] J.F.Ritt. Permutable rational functions. *Transactions of the American Mathematical Society*, Vol. 25, No. 3 (Jul., 1923), pp. 399-448.

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PROFESSOR JĀNIS MENCIS AND HIS SCIENTIFIC AND PEDAGOGICAL ACTIVITY

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Professor J. Mencis was a really talented mathematician and an outstanding math teacher who devoted all his talent and energy to creating a system of teaching mathematics in Latvia, its maintenance and continuous improvement. His talent of a mathematician and great capacity for work made him an outstanding methodologist. The entire Professor's research activity was closely linked to his practical teaching. The experience and knowledge gained from being a mathematics teacher served as the basis for theoretical generalisations about the teaching/learning process, its intensification, and improvement.

Professor's pedagogical activity was inextricably linked to his research in the field of teaching mathematics - since 1940 he participated in developing different textbooks in mathematics and methods of teaching mathematics and teaching aids. The first textbooks written by J. Mencis *Mathematics for Class 3 and Mathematics for Class 6* were issued in 1940 - 1943. Beginning with the year 1950, Jānis Mencis became the author of numerous school mathematics textbooks for junior and middle grades and the leading specialist in methods of teaching mathematics in Latvia. He carried out a large-scale research work to create a unified system of teaching mathematics, based on the most recent theoretical studies and findings in mathematics and pedagogical sciences.

Professor always advocated changes [1; 2], development, he was aware of the need for changes in mathematics education, as shown by the content of his textbooks and teaching resources. J. Mencis used to emphasise that changes come by themselves whether we are expecting them or not. The purpose of changes and the way of their implementation were significant for him, not only changes for the sake of changes.

The Professor's research and teaching activities could be conditionally divided into three phases: 1950s-1970s; 1970s - 1990s, and 2000 - 2011.

During the first research phase, Professor opposed to the formalism in teaching mathematics, in his publications and reports he analysed and explained the role of elementary exercises in promoting learners' understanding, relation of mathematics to practical life. Professor actively stood up for the great importance of mental arithmetic in developing learners' mathematical abilities, and still topical today is his saying: "Whatever you can, do it in your head!"

The second phase of J. Mencis' research activities could be associated with rather radical reforms of school curricula in the late 1960s and early 1970s, in the development and implementation of which Professor took an active part. Concurrently with the care for mathematics teacher training, J. Mencis focussed on training primary school maths teachers for teaching mathematics. He developed a methodological system for teaching mathematics in primary school.

In the year 1999 he finished his active teaching activities, but was still continuing his research and dedicated all his knowledge and energy to compiling mathematics textbooks and teaching/learning aids.

REFERENCES

[1] J. Mencis, seniors. Ieviešam jauno matemātikā: par TV raidījumu II kl. *Skolotāju Avīze*, 1969, 29.janv.

[2] J. Mencis, seniors. Ieviešam matemātikā jauno: [par televīzijas raidījumiem III un IV kl.]. *Skolotāju Avīze*, 1969, 1.okt.

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ON DIFFERENT TYPE SOLUTIONS OF THE DIRICHLET BVP

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We consider the Dirichlet problem

$$\begin{aligned} &x'' = f(t, x, x'),\\ &x(a) = A, \quad x(b) = B \end{aligned} \tag{1}$$

provided that all solutions are extendable to $[a, b]$. We define the type of a solution of BVP (1) as the number of zeros of $y(t)$ in (a, b), where $y(t)$ is a solution of the respective equation of variations

$$\begin{aligned} &y'' = f_x(t, x(t), x'(t))y + f_{x'}(t, x(t), x'(t))y'\\ &y(a) = 0, \quad y'(a) = 1. \end{aligned} \tag{2}$$

We claim that if there are two solutions of BVP (1) $u(t)$ and $v(t)$, $type(u) = m$, $type(v) = n$, $|m - n| \geq 2$, then there exist at last $|m - n| - 1$ more solutions of BVP (1).

REFERENCES

[1] M. Dobkevich and F. Sadyrbaev, *Types of solutions and approximation of solutions of second order nonlinear boundary value problems.* Proceedings of ICNAAM. Amer. Inst. Phys. Publ., **1168** (1), 260–263 (2009).

[2] M. Dobkevich. *On Construction of Converging Sequences to Solutions of Boundary Value Problems.* Mathematical Modelling and Analysis **15** (2), 189–197 (2010).

[3] M. Dobkevich. *Non-Monotone Approximations Schemes for Solutions of BVP for ODE.* Proceedings of IMCS of University of Latvia, **11**, 83–89 (2011).

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ON THE STABILITY OF SHALLOW MIXING LAYERS WITH NON-UNIFORM FRICTION[1]

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Linear stability analysis of shallow mixing layers is usually performed under the assumption that friction coefficient is constant in the whole region of the flow. There are cases where this assumption is not valid. One example is the flow during floods over partially vegetated area where friction force is not constant but varies considerably in the transverse direction. In the present paper spatial linear stability problem is solved for the case where friction coefficient is represented in the form $c_f(y) = c_0\gamma(y)$, where c_0 is constant and $\gamma(y)$ is a smooth differentiable shape function. Linear stability problem is solved numerically by means of a collocation method based ob Chebyshev polynomials. Spatial growth rates are calculated for different values of the parameters of the problem. Calculations are performed for the case where the friction coefficient varies monotonically from zero to some finite value. It is shown that in this case gowth rates for non-uniform friction are higher than for the case of constant friction.

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ON (L, M)-ROUGH SETS INDUCED BY MANY-VALUED L-RELATIONS

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We introduce the concept of an (L, M)-rough set, where L is a complete frame and M is an arbitrary complete lattice, and discuss basic properties of the category of (L, M)-rough sets and appropriately defined morphisms. Our special attention is paid to (L, M)-rough sets induced by many-valued L-fuzzy relations [1], which in essence are well-coordinated families of L-relations indexed by elements of complete lattices. In special case when $L = \{0, 1\}$ and $M = \{0, 1\}$ are two point lattices, "our" (L, M)-rough sets can be identified with classical Z. Pawlak's rough sets [2], while the case $L = [0, 1]$ and $M = \{0, 1\}$ lead us to the concept closely related to the notion of a fuzzy rough set, first introduced by D. Dubois and H. Prade [3] and further studied and generalized in various directions by different authors.

To develop the theory of (L, M)-rough sets and, specifically, to study them in the framework of the categories of approximate systems [4], [5], which proved to be useful specifically in the study of structures induced by L-relations, see e.g. [6], we introduce concepts of variable-range quasi-approximate and pseudo-approximate systems, which less restricted than the concept of a variable-range approximate systems. Specifically, it is important that the categories of variable-range quasi-approximate and pseudo-approximate systems include also (L, M)-rough sets induced by families of many-valued L-relations, which miss some of the properties of reflexivity, symmetry and transitivity or satisfy some weaken versions of these axioms.

REFERENCES

[1] Sang-Eon Han, In Soo Kim and A. Šostak. On approximate-type systems generated by L-relations Inf. Sci (submitted)

[2] Z. Pawlak. Rough sets International Journal of Computer and Information Sciences 11 p. 341-356, 1982

[3] D. Dubois, H. Prade. Rough fuzzy sets and fuzzy rough sets International Journal of General Systems 17 (2-3) p.191-209,1990

[4] A. Šostak. Towards the theory of M-approximate systems: Fundamentals and examples Fuzzy Sets and Syst. (161). p.2440-2461, 2010

[5] A. Šostak. Towards the theory of approximate systems: variable range categories Proceedings of ICTA2011, Islamabad, Pakistan. Cambridge University Publ., p.265-284, 2012

[6] A.Eļkins, A. Šostak/ On some categories of approximate systems generated by L-relations. Proceedings of Workshop on Rough Sets Theory p.14-19, 2011

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COPULAS AND MARKOV CHAINS

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Our paper deals with the estimation of copula-based semi parametric stationary Markov models [1]. In the problems of constructing a copula usually consider homogeneous Markov process and stationary Markov chain.

To describe the states of the Markov chain with distributions $\{X_s, X_u, ...\}$ a specially introduced the operator " $*$ " is used [2]. With denotation as C_{su} is a copula for (X_s, X_u), C_{ut} is a copula for (X_u, X_t) and C_{st} is a copula for (X_s, X_t) the following equation can be written

$$C_{ut} = C_{su} * C_{ut}, \quad s < u < t$$

In fact, the Kolmogorov - Chapman equation produces the Markov chain copula family [3]. If $\{(X_t), t \geq 0\}$ - Markov process with transition kernel $P_{st}(X_y)$ and marginal distributions $(F_t), t \geq 0$

$$C_{st}(F_s(x), F_t(y)) = P(X_s \leq x, X_t \leq y) = \int_{-\infty}^{x} P_{st}(u, (-\infty, y])\, dF_s(u)$$

However, if $(F_t), t \geq 0$ has uniform distribution, then one can conclude that Markov copula is the same as Markov transition density. On the other hand there is enough criticism of the use of copulas in the field of stochastic processes, in particular, an incomplete understanding of the term "temporal dependence" in the case of copula for a number of basic stochastic processes (the existence of ergodicity is broken, which does not quite match the initial process). For example, Andreas N. Lageräs the article [3] criticizes the earlier results for a number of commonly used copulas.

- Frechet copula generates strange Markov process - uniform and nonuniform Markov processes have the same probabilistic interpretation.

- Archimedean copula is not compatible with a dependency property of a Markov chain. This copula does not generate a Markov process with distributions $(X_1, X_2, ..., X_n)$ until while $(X_1, X_2, ..., X_n)$ are dependent.

- Built by Darsow [4] generalized homogeneous Markov copula does not cover all possible classes of homogeneous Markov chains.

These results will be discussed in our presentation.

REFERENCES

[1] D.B. Nelson, ARCH models as diffusion approximation. *J. of Econometrics*, **45**, 1990, 7–38.

[2] X. Chen and Y. Fan, Estimation of copula-based semiparametric time series models. *J. of Econometrics*, 2006.

[3] A.N. Lageräs. Copulas for Markovian dependence. *Bernoulli*, **16(2)**, 2010, 331–342.

[4] W.F. Darsow, B. Nguyen, E.T. Olsen. Copulas and Markov processes. *Illinois J.Math.*, **36(4)**, 1992, 600–642.

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STATIONARY SOLUTION OF MARKOV-SWITCHED GARCH(1,1) PROCESS

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Main results for stochastic differential equations apply to their stationary solutions. For obvious reasons underlying processes are simulated with stochastic difference equations. Such equations also have stationary solutions that depend on the time step h. In the current paper relation between statistical features of stationary solutions of stochastic difference and differential equations are investigated.

REFERENCES

[1] Daniel B. Nelson. ARCH models as diffusion approximations. Journal of Econometrics, 1990, vol. 45, issue 1-2, pages 7-38

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MOMENTS OF LINEAR MARKOV MAPPINGS

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Let $A(y), y \in \mathbb{Y} \subset \mathbb{R}$ be continuous $d \times d$-matrix-function and $\{y_t \in \mathbb{Y}, t \in \mathbb{Z}\}$ is given on filtered probabilistic space $(\Omega, \mathfrak{F}, \{\mathfrak{F}^t\}, \mathbf{P})$ homogeneous Markov chain with transition probability $p(y, dy)$, which satisfies Feller property [1]: $\forall v \in \mathbb{C}(\mathbb{Y}) :\ \ (\mathcal{P}v)(y) = \int\limits_{\mathbb{Y}} v(z)p(y, dz) \in \mathbb{C}(\mathbb{Y})$, exponential ergodic property: spectrum $\sigma(\mathcal{P})$ of operator $\mathcal{P}$ may be presented in a form $\sigma(\mathcal{P}) = \{1\} \bigcup \sigma_\gamma,\ \sigma_\rho \subset \{\lambda \in \mathcal{C} : |\lambda| < \gamma < 1\}$, and unique invariant measure $\mu(dy)$. Using this measure we may define Gilbert space $\mathbb{G} := \mathbb{L}_2^d(\mu)$ of square-integrable vector-functions with scalar product $< \mathbf{u}, \mathbf{v} >:= \int\limits_{\mathbf{Y}} (u(y), v(y))\mu(dy)$.Under assumption $\int_{\mathbb{Y}} ||A(y)||^2 \mu(dy) < \infty$ one may define on the space $\mathbb{G}$ linear continuous operator $\mathbf{A} : \mathbb{G} \to \mathbb{G}$: $\mathbf{v} \in \mathbb{C}_d(\mathbb{Y}), y \in \mathbb{Y} : (\mathbf{A}\mathbf{v})(y) := \int\limits_{\mathbb{Y}} A^T(z)\mathbf{v}(z)p(y, dz)$.

Lemma 1. *For all $s \in \mathbb{Z}$,$t \in \mathbb{N}$, $\mathbf{u} \in \mathbb{G}$, and $\mathbf{x} \in \mathbb{G}$*

$$\mathbf{E}\{(X_s^{s+t} x(y_s), v(y_{s+t}))/\mathfrak{F}^s\} = (\mathbf{x}(y_s), (\mathbf{A}^{\mathrm{t}}\mathbf{v})(y_s)) \tag{1}$$

where $\mathbf{X}_s^{s+t} := A(y_{s+t})A(y_{s+t-1}) \cdots A(y_{s+1})$.

This lemma permits to find all coordinates of the first moment $\mathbf{E}\{x_{s+t}(s, x, y)\} := \mathbf{m}_t(y)$ applying (1) to constant vectors $\mathbf{u} = \mathbf{e}_j, j = 1, 2, ..., d$ of the unit bases matrix $\mathbf{I} = \{\mathbf{e}_1, \mathbf{e}_2, ..., \mathbf{e}_d\}$:

$$\mathbf{m}_t(y) := \mathbf{E}\{x(s+t, s, x, y)\} := \mathbf{E}\{(X_s^{s+t} x/y_s = y\} = (\mathbf{A}^{\mathrm{t}}\mathbf{I})^T(y)x \tag{2}$$

In reality vector-function $\mathbf{m}_t(y)$ we can find only for finite phase space $\mathbb{Y}$. This paper proposes a working approximation $\bar{\mathbf{m}}_t$ of (2) for above defined Markov iterations with near to constant matrix $A(y, \varepsilon) = A_0 + \sum\limits_{k=1}^{m} \varepsilon^k A_k(y)$. The prposal algorithm is based on derived in [2] spectral projective operator decomposition for corresponding to matrix $A(y, \varepsilon)$ operator $\mathbf{A}(\varepsilon)$.

REFERENCES

[1] E.B. Dynkin: *Markov Processes*, NY, USA: Academic Press, 1965.

[2] V. Carkova and J. Goldsteine, On mean square stability of linear Markov difference equations, In *Proceedings of the 3rd International conference Aplimat, Slovak University of Technology, Bratislava* (2004), 313-319.

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ON OSCILLATORY NEHARI SOLUTIONS

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The Nehari theory deals in particular with superlinear differential equations of the form

$$x'' = -q(t)|x|^{2\varepsilon}x, \quad \varepsilon > 0. \tag{1}$$

The Nehari numbers $\lambda_n(a,b)$ are minimal values of the functional $H(x) = \frac{\varepsilon}{1+\varepsilon}\int_a^b x'^2(t)dt$ over the set of all $C^2([a,b],\mathbb{R})$ solutions of the boundary value problem (1),

$$x(a) = 0 = x(b), \quad x(t) \text{ has exactly } n-1 \text{ zeroes in } (a,b). \tag{2}$$

The BVP (1), (2) may have multiple solutions but not all of them are minimizers. Z. Nehari [1] posed the question is it true that there is only one minimizer associated with $\lambda_n(a,b)$. In [2] it was shown implicitly that there may be multiple minimizers associated with the number $\lambda_1(a,b)$. In the work [3] the example was constructed showing three solutions of the BVP (1), (2): two of solutions are non-even and one is an even function, besides two non-even solutions are minimizers.

Recently this problem was studied theoretically also by R. Kajikiya [4], [5].

In this talk we analyze Nehari numbers $\lambda_n(a,b)$ $(n > 1)$.

REFERENCES

[1] Z. Nehari. Characterictic values associated with a class of nonlinear second-order differential equations. *Acta Math.*, **105** (3), 1961, 141–175.

[2] F. Sadyrbaev. Solutions of an Emden-Fowler equation. *Differential Equations, Kluwer*, **25** (2), 1989, 560–565.

[3] A. Gritsans and F. Sadyrbaev. Characteristic numbers of non-autonomous Emden-Fowler type equations. *Math. Modelling and Analysis*, **11** (3), 2006, 243–252.

[4] R. Kajikiya. Non-even last energy solutions of the Emden-Fowler equation. *Proc. Amer. Math. Soc.*, **140** (4), 2012, 1353–1362.

[5] R. Kajikiya. Non-even positive solutions of the one-dimensional p-Laplace Emden-Fowler equation. *Appl. Math. Lett.*, **25** (11), 2012, 1891–1895.

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SPECTRAL MIXTURE ANALYSIS OF MULTITEMPORAL MEDIUM RESOLUTION SATELLITE IMAGES

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Analysis of medium resolution satellite images provides the opportunity to acquire regular information about land cover in vast areas. Each pixel in satellite image represents area on the ground by single measurement in multispectral bands, while in case of medium resolution images this area often comprises multiple land cover types (such pixels are called mixed pixels). Well known method for land cover estimation at subpixel level is Spectral Mixture Analysis (SMA) [1].

Mixed pixel reflectance r can be described as a linear combination of spectral signatures for pure land cover types (called endmembers) weighted by their subpixel fractional cover:

$$r = Mf + \varepsilon, \tag{1}$$

where M is a matrix in which each column contains reflectance spectrum of one endmember (pure land cover type), f - column vector containing fractional coverage of each land cover type within a pixel, ε - difference between observed pixel reflectance r and reflectance computed from the model. If the number of endmember types is less than number of multispectral bands, then overdetermined system of equations can be solved to minimize ε. Successful application of SMA depends on the selection of endmembers. Since pixel reflectances for specific image depends not only on land cover type, but also other factors, effective SMA of multitemporal images becomes a challenge.

The aim of this study is to evaluate the application of SMA to multitemporal satellite images with index based selection of endmembers for monitoring of vegetation fractional covers and to investigate temporal effects on SMA results.

Application of SMA was evaluated for Landsat TM summer and SPOT HRIVIR 1 and 2 winter images comprising the area of North Kurzeme. Three endmember classes were established: vegetation, soil/snow and shade. Spectrum of shade was modelled, while automated methodology was applied for the selection of vegetation and soil/snow endmembers. Multiple endmember unconstrained SMA were applied to the images.

Results of vegetation fractional coverage were compared between summer images of different years and between summer and winter images of 2011 to investigate stability of fractional coverage estimates. Physical meaningfulness of the estimates were checked using regular forest inventory (RFI) data base, which contains estimates and measurements of stand level forest inventory parameters.

REFERENCES

[1] B. Somers, G.P. Asner, L. Tits and P. Coppin. Endmember variability in spectral mixture analysis: A review. *Remote Sensing of Environment*, **115.7** 1603-1616, 2011.

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NEW MATHEMATICAL METHODS FOR FINANCIAL TIME SERIES FORECASTING

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Financial time series forecasting is complicated task with many practical applications. From forecasting point of view financial time series P(t) consist of three different components

$$P(t) = \alpha R(t) + \beta H(t) + \gamma S(t), \alpha + \beta + \gamma = 1, \tag{1}$$

where regular part $R(t)$ is determined by superposition of different scale periodic processes (e.g. economic cycles or seasonal price oscillations), chaotic part $H(t)$ is determined by properties of economy as chaotic dynamic system, stochastic part $S(t)$ is determined by random factors. Depending on weights α , β, γ values time series can be or completely predictable ($\alpha = 1$), or completely unpredictable ($\gamma = 1$), or be partly predictable.

To investigate possibilities to predict time series we generate time series as specifically prepared mixtures of regular, chaotic and stochastic components with given α, β and γ, presented by superposition of several Van der Pol oscillators, Lorenz attractors and random generators. Such "oscillation factory" can generate wide set of time series with given statistical properties and different difficulty degrees for forecasting. Regular component $R(t)$ is predictable best of all, but on highly competitive financial markets contributions of regular oscillations are very small ($0.5 - 3\%$). Stochastic component $S(t)$ is completely unpredictable in the case of repeated coin throws the previous history does not influence the result of next throw and there is no sense to analyse it for forecasting. Chaotic component predictability lies between $R(t)$ and $S(t)$.

The main task of investigation is to work out such mathematical methods of time series analysis, which can extract weak regular signal from mixture signal+noise and use it for financial time series forecasting. The development of such methods of forecasting is very important for practical applications in automated investment system creation and portfolio management. Such methods as singular spectral analysis (SSA) and detrended fluctuation analysis (DFA) and different discrete time filtering procedures are used for α, β and γ parameters evaluation and signal recognition.

REFERENCES

[1] N. Golyandina, V. Nekrutkin and A. Zhigljavsky. Analysis of Time Series Structure: SSA and related techniques. *Chapman & Hall*, **CRC Press** 2001.

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ON THE MATHEMATICAL MODELLING OF MHD CHANNEL FLOW AND TEMPERATURE[1]

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In [1] the flow of the viscous electrically conducting incompressible liquid around infinite periodically placed cylinders is considered. As simple model in this talk we examine the MHD channel flow between two infinite planes $D = [x \in [0,l], y \in (0,L), z \in R]$ with the velocity vector $\vec{U} = U_0(u(y),0,0)$ in the x-direction, where L is the half width of the channel and l is the period of the flow in x direction. We analyze the 2D flow with convection and temperature in uniform transversal external magnetic field $\vec{B} = (0, B_0, 0)$ depending on direction of the gravitation $\vec{g} = g\sin(\beta), -g\cos(\beta,0), \beta = 0; \pm\pi/2$. The liquid motion can be described by the dimensionless (the characteristic value are L, U_0, B_0) stationary 1D MHD (ODEs of 4-th order) and heat transfer (ODEs of second order) equations [2]:

$$\begin{cases} \Psi^{(4)}(y) = Ha^2\Psi''(y) - \frac{Gr}{Re}T'(y)\sin(\beta), \\ T''(y) = -K_T(E_z + \Psi'(y))^2 \end{cases} \tag{1}$$

where Ψ, T are the stream function $(u(y) = \Psi'(y))$ and temperature, Ha, Re, Gr are are Hartman, Reynolds and Grashof numbers, K_T is the heat source parameter, $E_z = -1/l$ is the azimuthal component of the electric field (the walls of the channel are electrically non-conducting), β is the angle between the (-Oy) axes and direction of the gravitation vector. We have following boundary conditions by $y = 0$ (the wall) and $y = 1$ (the symmetric axis): $T(0) = 0, T(1) = 1, \Psi(0) = \Psi'(0) = 0, \Psi(1) = 1, \Psi''(1) = 0$. In this talk we analyze numerically the MHD flow and temperature depending on different values of the parameters. $Gr = 0; 1000; 2000; 3000; 5000; 10000, Re = 100, Ha = 0; 10; 100, K_T = 0; 1$. If $K_T = 0$ then the distribution of the teperature is linear $T(y) = 1-y, T'(y) = -1$ and we can solve the problem analytically. If $Ha = 0$ then $u(y) = \Psi'(y) = \frac{g}{6}y^3 - (1.5 - \frac{5g}{16})y^2 + (3 + \frac{g}{8})y$, where $g = \frac{Gr}{Re}\sin(\beta)$ (by g=0 we have the Poiseuile profile). For $Ha \neq 0 : u(y) = C_2 + Ha(C_3\cosh(Hay) + C_4\sinh(Hay) - \frac{g}{Ha^2}y$, where $C_2 = -HaC_3$; $C_3 = (\frac{g}{Ha^4}(\cosh(Ha) - 1) - (1 + \frac{g}{2Ha^2})\sinh(Ha))/d$; $C_4 = -(\frac{g}{Ha^4}(\sinh(Ha) - Ha) - (1 + \frac{g}{2Ha^2})\sinh(Ha))/d$, $d = Ha\cosh(Ha) - \sinh(Ha)$(by g=0 we have the Hartman profile).The pressure by y=1 $\tilde{p}(x) = p(x) + 0.5u(1)^2$ can obtained from the first Navier-Stokes equation $Re\frac{\partial\tilde{p}(x)}{\partial x} = \Psi'''(1) - Ha^2((E_z + \Psi'(1)) + gT(1), x = \overline{0,l}, p(0) = 0$.

REFERENCES

[1] H. Kalis, M. Marinaki and A. Gedroics. Mathematical modelling of the 2D MHD flow around infinite cylinders with square section placed periodically. *Magnetohydrodynamics -MHD*, **48** 3.2012,243-258

[2] G.K. Batchelor. An introduction to fluid dynamics. *Cambridge the University Press*, **1970** .

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ON MATHEMATICAL MODELLING OF MASS TRANSFER IN THE MULTI-LAYERED DOMAIN

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The process of diffusion in 3-D multi-layered medium Ω_i is considered,

$$\Omega_i = \{(x, y, z) : x \in (0, l), y \in (0, L), z \in (z_{i-1}, z_i)\}, i = \overline{1, N}, z_0 = 0, z_N = Z.$$

We discuss the solving of 3-D boundary-value problem with boundary conditions of the 3rd type in the N-layered domain of homogeneous materials for elliptic type partial differential equations of second order with piece-wise diffusion coefficients

$$D_{ix}\partial^2 c_i/\partial x^2 + D_{iy}\partial^2 c_i/\partial y^2 + D_{iz}\partial^2 c_i/\partial z^2 + f_i(x, y, z) = 0, i = \overline{1, N},$$

here D_{ix}, D_{iy}, D_{iz}, are constant diffusions cefficients, $c_i = c_i(x, y, z)$ - the concentrations of metals in every layer, $f_i(x, y, z)$ - the fixed source function.
The values c_i and the flux functions $D_{iz}\partial c_i/\partial z$ must be continuous on the contact lines between the layers $z = z_i$, therefore the coherence conditions on the contact lines between the layers are given. The layered material is bounded below and above with the plane surfaces $z = 0, z = Z$ with fixed boundary conditions in following form:

$$D_{1z}\frac{\partial c_1(x, y, 0)}{\partial z} - \alpha_1(c_1(x, y, 0) - C_0(x, y)) = 0,$$

$$D_{Nz}\frac{\partial c_N(x, y, Z)}{\partial z} + \alpha_2(c_N(x, y, Z) - C_a(x, y)) = 0,$$

where C_0, C_a are given concentration-functions, $\alpha_1 > 0$, $\alpha_2 > 0$ are the mass transfer coefficients. We apply the averaging [2] and finite difference method for solving the corresponding boundary-value problem. As opposed to the models analyzed previously [1], the newly established mathematical model has the boundary conditions of the 3rd type on both plane surfaces $z = 0, z = Z$ thus envisages modelling mass transfer through the top layer and also towards the Earths interior.

REFERENCES

[1] H. Kalis, I. Kangro and A. Gedroics. Numerical solution of some diffusion problems in 3-layered 3D domain. *Journal of Mathematics and System Science*, **Volume 3, Number 6, June 2013,** David Publishing Company, Rosemead, CA 91770, USA.2013, 309-318

[2] A. Buikis. The approximation with splines for problems in layered systems. *Riga, Acta Universitatis Latviensis* , **592** 1994.135-138, in Latvian

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MODELING AND PARALLEL MONTE CARLO SIMULATION OF SPIN SYSTEMS

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Systems of many interacting spins are considered, where the spin is an n–component vector. Such systems are often described by lattice spin models with certain Hamiltonian $\mathcal{H}$ of the form

$$\frac{\mathcal{H}}{k_B T} = -\beta \left(\sum_{\langle ij \rangle} \mathbf{s}_i \mathbf{s}_j + \sum_i \mathbf{h} \mathbf{s_i} \right) , \tag{1}$$

where T is temperature, k_B is the Boltzmann constant, $\mathbf{s}_i$ is the spin variable (n–component vector of unit length) of the ith lattice site, β is the coupling constant, and $\mathbf{h}$ is the external field with magnitude $\mid \mathbf{h} \mid = h$. The summation takes place over all pairs $\langle ij \rangle$ of the nearest neighbors in the lattice. The model with $n = 1$ is the Ising model, whereas those with $n \geq 2$ are called the $O(n)$ models owing to the $O(n)$ global rotational symmetry at $\mathbf{h} = \mathbf{0}$. We are interested also in soft-spin models, such as the φ^4 model, where the modulus of spin variable is not fixed. Equilibrium properties of these lattice spin models are given by the Boltzmann statistics, according to which each spin configuration has a statistical weight $\propto \exp\left(-H/(k_B T)\right)$. Monte Carlo simulation is a powerful tool to generate this equilibrium distribution and thus compute various average quantities, describing the equilibrium properties.

We are particularly interested in Monte Carlo analysis of critical phenomena and Goldstone mode singularities in n-vector models to verify several challenging analytical predictions. Efficient parallel Monte Carlo algorithms have been developed and lattices of very large sizes have been simulated for this purpose [1; 2; 3]. We report the results of our Monte Carlo analysis for the 3D Ising model with linear lattice sizes up to $L = 2048$, as well as the results for $O(n)$ models with $n = 2, 4, 10$ and some recent results for the 2D φ^4 model.

REFERENCES

[1] J. Kaupužs, J. Rimšāns, R. V. N. Melnik. Parallelization of the Wolff single-cluster algorithm. *Phys. Rev. E*, **81** Article Number 026701, 2010.DOI:10.1103/Phys.RevE.81.026701

[2] J. Kaupužs, J. Rimšāns, R. V. N. Melnik. Scaling regimes and the singularity of specific heat in the 3D Ising model. *Commun. Comput. Phys.*, **14** (2), pp. 355-369.doi:10.4208/cicp.240512.120912a

[3] J. Kaupužs, J. Rimšāns, R. V. N. Melnik. Correlation functions, universal ratios and Goldstone mode singularities in n-vector models. *Commun. Comput. Phys.*, **15** (5), pp. 1407-1430.doi:10.4208/cicp.260613.301013a

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ON BOUNDARY VALUE PROBLEM FOR CUBIC-QUADRATIC NONLINEARITIES

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In this talk we first consider the problem

$$x'' = -x + b\,x^3, \quad x(0) = 0,\ x(T) = 0, \tag{1}$$

which may have only finite number of solutions for $b > 0$.

On the other hand the problem

$$x'' = -x + b\,x^2, \quad x(0) = 0,\ x(T) = 0, \tag{2}$$

may have also only finite number of solutions for $b > 0$.

We also consider the non-autonomous problem

$$x'' = -x + \alpha(t)x^2 + \beta(t)x^3, \quad x(0) = 0,\ x(T) = 0, \tag{3}$$

where α and β are step-wise functions [1]

$$\alpha(t) = \begin{cases} 0, & t \in I_{2i-1} \\ b, & t \in I_{2i} \end{cases} \tag{4}$$

$$\beta(t) = \begin{cases} 0, & t \in I_{2i} \\ b, & t \in I_{2i-1}, \end{cases} \tag{5}$$

where $\bigcup_{i=1}^{n} I_i = [0, T]$. Therefore trajectories of equation $x'' = -x + \alpha(t)x^2 + \beta(t)x^3$ switch from trajectories of (2) to trajectories of (1) and vice versa. We consider multiplicity of solutions of problem (3).

REFERENCES

[1] E. Ellero and F. Zanolin. Homoclinic and heteroclinic solutions for a class of second-order non-autonomous ordinary differential equations: multiplicity results for stepwise potentials.. *Boundary Value Problems,*, **2013, 2013:167,** http://www.boundaryvalueproblems.com/content/2013/1/167.

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ON A SPECIAL CASE OF BIRKHOFF INTERPOLATION

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Suppose we are given natural numbers n and N, a grid Δ, a set M and a vector Y

$$\Delta = \{t_1;\ t_2;\ \ldots;\ t_n \in \mathbb{R}\ :\ t_1 < t_2 < \ldots < t_n\};$$

$$M = \{(i_j, k_j) \in \overline{1,n} \times \overline{0, N-1}\ :\ j \in \overline{1,N}\};$$

$$Y = (y_{i,k})_{(i,k)\in M}.$$

Consider the polynomial space P_{N-1}, i.e., the space of polynomials of the degree less than N. We deal with the following interpolation problem: find a $p \in P_{N-1}$ satisfying

$$\forall i \in \overline{1,n}\ \forall k \in \overline{0, N-1} \quad (i,k) \in M \Rightarrow p^{(k)}(t_i) = y_{i,k}. \tag{1}$$

This is known as the classical Birkhoff (or lacunary) interpolation by polynomials (see, e.g., [1], [2]). In contrast to Hermite and Lagrange interpolation problems, the interpolation polynomial may not exist or there can be several polynomials satisfying (1).

The main attentions of this talk is devoted to a special case of the Birkhoff interpolation problem, when $(i,k) \in M \Rightarrow k \in \{0, 1\}$, i.e., the constraints for the interpolant are imposed only on its values and the first order derivatives. When problem (1) does not have a solution, we seek a polynomial of degree higher than $N-1$ which satisfies (1).

REFERENCES

[1] G. G. Lorentz, K. Jetter and S. D. Riemenschneider. *Birkhoff interpolation*, Cambridge University Press, Cambridge, 1984.

[2] Y. G. Shi. *Theory of Birkhoff interpolation*, Nova Science Publishers, Hauppauge, New York, 2003.

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BOUNDARY EFFECTS ON PARTICLE MIGRATION IN STOKES FLOW

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Boundary effects in particulate suspensions have been studied in various contexts e.g. for macromolecular solutions [1], in magnetorheology, hemorheology, studies of particle sedimentation etc. When the Reynolds number is low, Stokes approximation can be used. The hydrodynamic interactions of particles mediated by the fluid with viscosity η can be studied using the fundamental solution of Stokes equation

$$0 = -\nabla p + \eta \nabla^2 v + \delta(r - r_0)F, \qquad \nabla \cdot v = 0, \tag{1}$$

which is $v = S \cdot F$ with the Oseen tensor (stokeslet)

$$S_{ij}(r) = \frac{1}{8\pi\eta}\left(\frac{I}{|R|} + \frac{R^2}{|R|^3}\right), \quad R = r - r_0.$$

Rotating particles can be studied in a similar fashion by approximating the presence of the particle by a force couple.

We review the applications of Green function technique for incompressible Stokes equations [2]. We demonstrate the interactions among several particles suspended in Newtonian liquid near a solid wall for various particle configurations by using analytical solution of the Stokes equation. The results give qualitative and quantitative insight in the particle migration behaviour, namely, the way the particles migrate away from the boundary.

REFERENCES

[1] H. Ma and M.D. Grahama. Theory of shear-induced migration in dilute polymer solutions near solid boundaries. *Phys Fluids*, **17** Article Number 083103 (2005).

[2] J. Ainley, S. Durkin, R. Embid, P. Boindala, R. Cortez. The method of images for regularized Stokeslets. *J Comp Phys*, **227** 4600–4616 (2008).

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APPLICATION OF STATISTICAL CLASSIFIER FOR THE PRODUCTION OF POPULATION STATISTICS

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The aim of the speech is to present the recently developed methodology for the production of population statistics [1]. The methodology is developed by the Central Statistical Bureau of Latvia. We see it as an innovative approach in the field of official population statistics.

Population statistics were compiled directly from the State Population Register data previously. Unfortunately the precision of the recent population statistics produced this way was not satisfactory enough. The precision deficiency was clearly shown by the results of the latest population census 2011. The methodological work was started in 2012 to develop a new methodology that would produce more precise population statistics.

There are 2.2 million individuals registered as the residents of Latvia according to the Population Register data. The problem is that not all of them are the residents of Latvia according to the usual residence definition used for European population statistics [2]. The idea for the new methodology was to use binary statistical classifier to split the set of individuals into two subsets – residents of Latvia according to the usual residence definition and residents of other countries.

The latest population census data and data from several administrative registers were merged at individual level. The merged data were used to build binary statistical classifier. Several classification methods were tested, for example, logistic regression, generalized boosted regression, support vector machines, regularized discriminant analysis, neural networks. More details regarding the development of the methodology will be given during the speech.

The developed methodology has been approved and it is implemented in the statistical production. The annual population statistics for 2012 and 2013 has been produced using the new methodology. The main figures are shown in table 1.

Year	Total	Males	Females
2012	2 044 813	934 812	1 110 001
2013	2 023 825	926 580	1 097 245

Table 1.
ISG021. Population by Sex at the Beginning of Year, source: Central Statistical Bureau of Latvia

REFERENCES

[1] Central Statistical Bureau of Latvia. Method used to produce population statistics. *Methodological documentation*, Riga, 2014. Retrieved from http://www.csb.gov.lv/en/statistikas-temas/metodologija/number-population-and-key-demographic-indicators-36808.html

[2] The European Parliament and the Council of the European Union. Regulation (EU) No 1260/2013 on European demographic statistics. *Official Journal of the European Union*, **56** L 330, 10 December 2013, p. 41. http://eur-lex.europa.eu/legal-content/EN/TXT/?uri=CELEX:32013R1260

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ON 2D MODELS IN DESIGN OPTIMIZATION FOR MAGNETIC GEAR[1]

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Magnetic gears are widely used in mechanical devices and machines nowadays. Corresponding design optimization tasks are of large interest.

As an example, we consider the mechanism involving rectangle-shaped magnets and its two dimensional model first proposed in [1].

We consider the equations of magnetostatics in form (1):

$$\begin{cases} \nabla \times \mathbf{H} = 0, \\ \nabla \cdot \mathbf{B} = 0, \\ \mathbf{B} = \mu(\mathbf{H} + \mathbf{M}), \end{cases} \tag{1}$$

where $\mathbf{B}$ is the magnetic flux density, $\mathbf{H}$ is the magnetic field intensity, $\mathbf{M}$ is the magnetization vector, and μ is the material's magnetic permeability.

We develop a 2D vector-potential formulation and get the numerical finite element solution.
Simulations are made in MATLAB and FEMM; we calculate the field $\mathbf{B}$ and get some insight for extrema of transmitted torque functional in different situations.

Further we analyze the task depending on three parameters : α, h, n: respectively angular displacement of the driver in magnetic gear, height of a permanent magnet and the number of magnetic pole pairs.

By using PSO technique (particle swarm optimization), we maximize the functional $F(\mathbf{B}, \alpha, h, n)$, where F is the transmitted torque.
The convergence of this method is obtained. So is the parameters' corresponding optimal design.

In order to reduce computational time, mesh moving methods are also discussed. Such approach demands the construction of auxiliary linear elasticity problems for each given set of parameters [2].

REFERENCES

[1] Yi-Chang Wu and Chih-Wen Wang. Transmitted Torque Analysis of a Magnetic Gear Mechanism with Rectangular Magnets. , Appl. Math. Inf. Sci, 2013.

[2] Shuangxia Niu, Ningning Chen, S. L. Ho, and W. N. Fu. Design Optimization of Magnetic Gears Using Mesh Adjustable Finite-Element Algorithm for Improved Torque. *IEEE TRANSACTIONS ON MAGNETICS*, **48** NO.11, 2012.

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APPROXIMATION OF AGGREGATION OPERATORS BASED ON FUZZY EQUIVALENCE RELATION[1]

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Our talk deals with the problem of approximation of general aggregation operators taking into account a fuzzy equivalence relation. Upper and lower approximation operators are considered as generalizations of factoraggregation introduced in [1] and developed in [2].

Let $A\colon [0,1]^n \to [0,1]$ be an aggregation operator, T be a left continuous t-norm, $\overrightarrow{T}$ be the residuum of T and E be a T-fuzzy equivalence relation defined on the universe D. We consider the general aggregation operator $\tilde{A}\colon ([0,1]^D)^n \to [0,1]^D$, which is the pointwise extension of A. Upper and lower approximation operators $\tilde{A}_{E,T}$ and $\tilde{A}_{E,\overrightarrow{T}}$ are defined respectively by

$$\tilde{A}_{E,T}(\mu_1,\mu_2,\ldots,\mu_n)(x) = \sup_{u\in D} T(E(x,u), A(\mu_1(u),\mu_2(u),\ldots,\mu_n(u))) \tag{1}$$

and

$$\tilde{A}_{E,\overrightarrow{T}}(\mu_1,\mu_2,\ldots,\mu_n)(x) = \inf_{u\in D} \overrightarrow{T}(E(x,u)|A(\mu_1(u),\mu_2(u),\ldots,\mu_n(u))), \tag{2}$$

where $x \in D$ and $\mu_1,\mu_2,\ldots,\mu_n \in [0,1]^D$ are fuzzy sets.

We investigate properties of these approximation operators and illustrate our approach by numerical examples.

REFERENCES

[1] P. Orlovs, O. Montvida and S. Asmuss: *An analysis of bilevel linear programming solving parameters based on factoraggregation approach.* Aggregation Functions in Theory and in Practise: Proceedings of the 7th International Summer School on Aggregation Operators, Advances in Intelligent Systems and Computing, Springer-Verlag, **228** (2013) 345–354.

[2] P. Orlovs, O. Montvida and S. Asmuss: *A choice of bilevel linear programming solving parameters: factoraggregation approach.* Proceedings of the 8th conference of the European Society for Fuzzy Logic and Technology, Advances in Intelligent Systems Research, Atlantis Press, **32** (2013) 489–496.

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ON MINIMAL RIGHT-ANGLED TRIANGLE INCLUDING FIXED SUPERELLIPSE

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Let $S(F)$ be the area of a plane figure F. The problem: *Can convex figure F be included in such a right-angle triangle T that $S(T) \leq 2 \cdot S(F)$* is considered there. It is rather easy to state the estimation $S(T) \leq 2 \cdot S(F)$ for triangles and parallelograms. In [1] this problem has been solved positively for trapeziums. Some similar problems have been formulated and solved in the book [2].

The attention is paid to superellipses

$$E(a,b,n) := \left\{(x,y) : \left|\frac{x}{a}\right|^n + \left|\frac{y}{b}\right|^n \leq 1\right\},\ a,b,n > 0.$$

$E(a,b,n)$ is the convex shape for $n \geq 1$. For $n \geq 1.4$ it will be proved the estimation

$$S(T) < \left(2 - \frac{0.1}{n+3}\right) S(E).$$

REFERENCES

[1] Werner M., Über Trapezen umbeschriebene rechtwinklige Dreiecke, Wiss. Beitr. M.-Luther-Univ., Halle-Wittenberg, 1989, Nr. 56, 161-170.

[2] Shklyarsky D. O., Chentsov N. N., Yaglom I. M., Geometrical Estimations and Problems from Combinatorial Geometry, Moscow, Nauka, 1970, 384 p. (In Russian)

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EMPIRICAL LIKELIHOOD METHOD OF SURVIVAL DATA IN TWO-SAMPLE CASE

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Empirical likelihood (EL) method is widely used to make nonparametric inference on different parameters since it has been introduced by Owen ([1], [2]) for the independent data. However, the nonparametric likelihood was first mentioned already by Thomas and Grunkemeier (1975) in the context of survival analysis to obtain better confidence intervals involving Kaplan-Meier estimator.

For a long time there was no common approach how to use the EL method for other survival parameters. The adjusted empirical likelihood method for censored data was proposed by Wang and Jing ([3]), allowing to construct the confidence intervals for a class of functionals of a survival function. Later this approach has been shown as a special case of plug-in empirical likelihood method.

The extension of the plug-in empirical likelihood method for two-sample problems has been studied by Valeinis ([4]). The limiting distribution for the adjusted EL method in two-sample case has been derived and the implementation of the method was made to analyze some real data examples.

REFERENCES

[1] A. B. Owen. Empirical likelihood ratio confidence intervals for a single functional. *Biometrika*, **75** (2):237–249, 1988.

[2] A. B. Owen. Empirical likelihood ratio confidence regions. *The Annals of Statistics*, **18** (1):90–120, 1990.

[3] Q. H. Wang and B. Y. Jing. Empirical likelihood for a class of functionals of survival distributions with censored data. *Annals of the Institute of Statistical Mathematics*, **53** (3):517–527, 2001.

[4] J. Valeinis. *Confidence bands for structural relationship models*. Phd thesis, Goerg-August-University of Goettingen, Goettingen, 2007.

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ASYMPTOTIC ANALYSIS OF MARKET MODEL WITH CONTROLLED BY EXCESS RETURN HETEROSCEDASTIC VOLATILITY

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Our paper deals with simple market model [1] defined by increment for the log of cumulative excess returns Y_t on a portfolio $Y_{t_{k+1}} - Y_{t_k} = hc\sigma^2_{t_{k+1}} + \sqrt{h}\sigma_{t_{k+1}} Z_{t_{k+1}}$ with volatility $\{\sigma^2_t\}$ given by GARCH(1,1) process $\sigma^2_{t_{k+1}} = h\omega + \left(1 - \alpha\sqrt{h} - h\theta\right)\sigma^2_{t_k} + \alpha\sigma^2_{t_k}\sqrt{h}Z^2_{t_{k+1}}$ where $\{t_k, k = 0, 1, 2, ...\}$ is uniform partition of length $t_{k+1} - t_k = h$, and market uncertainty $\{Z_t\}$ is i.i.d. $N(0, 1)$ sequence. Taking the length of time intervals between observations h more and more finely the author of paper [1] derives the stochastic approximation for $\{Y_t, \sigma_t\}$ as the system of stochastic differential Ito equations. However, as it has been noted in many devoted to market analysis papers (see, for example, [2]), market uncertainty preferably should be induced as correlated time series. Besides switching time moments t_k in many market models may be also such random variables that permit analyze the system Y_t, σ^2_t not as discrete time series but as compound Poisson process [3]. Therefore in a difference of the paper [1] we suppose the market uncertainty as AR(1) process $Z_{t_{k+1}} = \rho Z_{t_k} + \sqrt{1-\rho^2}\xi_{t_{k+1}}$, where $\{\xi_{t_k}\}$ is i.i.d. $N(0, 1)$ sequence and assume that the ranges $\{t_k - t_{k-1},\ \ k \in \mathbb{N}\}$ are exponentially distributed $\mathbb{P}\{t_{k+1} - t_k > t/\mathfrak{F}^{t_k}\} = \exp\left\{-\frac{g(Y_{t_k})}{h}t\right\}$ where $\mathfrak{F}^t$ is minimal sigma-algebra defined by $\{\xi_{t_k}, t_k \le t\}$. This model we may interpret as the impulse dynamical system with fast Markov switching and for asymptotic analysis with $h \to 0$ to apply the limit theorem derived in the paper [4]. These assumptions lead to excess returns-volatility relation given by system of stochastic Ito equations:

$$
\begin{aligned}
dY(t) &= cg(y)\sigma^2 + \sigma\sqrt{g(y)\frac{1+\rho}{1-\rho}}dw_1(t) \\
d\sigma^2(t) &= g(y)\left(\omega - \sigma^2\left(\theta - 2\alpha^2\frac{\rho^2}{1-\rho^2}\right)\right)dt + \sigma^2\alpha\sqrt{\frac{2g(y)(1+\rho^2)}{1-\rho^2}}dw_2(t)
\end{aligned}
$$

which will be discussed in our presentation.

REFERENCES

[1] D.B. Nelson, ARCH models as diffusion approximation.*J. of Econometrics*, **45**, 1990, 7–38.

[2] J. Peck and K. Shell, Market Uncertainty: Correlated and Sunspot Equilibria in Imperfectly Competitive Economies. *Imperfectly Competitive Economies Review of Economic Studies*, **58**, 1991, 1011–1029.

[3] X. Zhang, T. Kuen Siu, and Q. Meng, Portfolio Selection in the Enlarged Markovian Regime-Switching Market.*SIAM Journal of Control and Optimization*, **48(5)**, 2010, 3368–3388.

[4] A.V. Skorokhod *Asymptotic Methods in the Theory of Stochastic Differential Equations*,AMS, 2009.

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CONJUGACY AND ASYMPTOTIC EQUIVALENCE OF IMPULSIVE DIFFERENTIAL EQUATIONS IN BANACH SPACE[1]

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Consider the following system of impulsive differential equations in Banach space $\mathbf{X} \times \mathbf{Y}$:

$$\begin{cases} dx/dt &= A(t)x + f(t,x,y), \\ dy/dt &= B(t)y + g(t,x,y), \\ \Delta x\big|_{t=\tau_i} &= x(\tau_i+0) - x(\tau_i-0) = C_i x(\tau_i-0) + p_i(x(\tau_i-0), y(\tau_i-0)), \\ \Delta y\big|_{t=\tau_i} &= y(\tau_i+0) - y(\tau_i-0) = D_i y(\tau_i-0) + q_i(x(\tau_i-0), y(\tau_i-0)), \end{cases} \tag{1}$$

satisfying the *conditions of separation*

$$\nu = \max\left(\sup_s \left(\int_{-\infty}^{s} |Y(s,t)||X(t,s)|\,dt + \sum_{\tau_i \le s} |Y(s,\tau_i)||X(\tau_i-0,s)|\right),\right.$$

$$\left.\sup_s \left(\int_{s}^{+\infty} |X(s,t)||Y(t,s)|\,dt + \sum_{s<\tau_i} |X(s,\tau_i)||Y(\tau_i-0,s)|\right)\right) < +\infty,$$

and $f(t,\cdot), g(t,\cdot), p_i, q_i$ are ε-Lipshitz, $f(t,0,0) = p_i(0,0) = 0$, $g(t,0,0) = q_i(0,0) = 0$.

Using nonexponential Green type map we find a simpler system of impulsive differential equations that is conjugated and asymptotic equivalent to the given one. Using this result we obtain sufficient conditions that invertible and noninvertible system (1) is asymptotic equivalent to the linear system

$$\begin{cases} dx/dt &= A(t)x, \\ dy/dt &= B(t)y, \\ \Delta x\big|_{t=\tau_i} &= C_i x(\tau_i-0), \\ \Delta y\big|_{t=\tau_i} &= D_i y(\tau_i-0), \end{cases} \tag{2}$$

in the case when ε depends on t and tends to zero as $t \to +\infty$ sufficiently rapidly.

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THE EXISTENCE RESULTS FOR SOME NONLINEAR BOUNDARY VALUE PROBLEM[1]

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The existence results are established for the nonlocal boundary value problem

$$x'' = -\mu x^+ + \lambda x^- + h(t, x, x')$$

$$x'(0) = 0, \quad \int\limits_0^1 x(s)ds = 0$$

provided that $h(t, x, x')$ is continuous and Lipschitzian in x and x'.

The results are based on the study of spectrum for the problem

$$x'' = -\mu x^+ + \lambda x^- \quad x'(0) = 0, \quad \int\limits_0^1 x(s)ds = 0.$$

REFERENCES

[1] N. Sergejeva. On the Fučík type problem with the integral and Neumann conditions. *Proc. of IMCS of University of Latvia*, **13,** 98–103, 2013.

[1] This work was partially supported by the grant 345/2012 of the Latvian Council of Science

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ON THE TRANSFORMATION OF THE THIRD ORDER DIFFERENTIAL EQUATIONS INTO THE TWO-DIMENSIONAL SYSTEMS

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Third order differential equations which can be transformed by the change of variables into the two-dimensional systems are considered. Connection between the behavior of the solutions of equations and the behavior of trajectories of systems on the phase plane is studied. Some properties of the solutions are interpreted in terms of the trajectories of the phase portrait.

REFERENCES

[1] I.V. Astashova. Application of Dynamical Systems to the Study of Asymptotic Properties of Solutions to Nonlinear Higher-Order Differential Equations. *Journal of Mathematical Sciences. Springer Science+Business Media*, 126(2005), 1361–1391.

[2] W.A. Coppel. A Survey of Quadratic Systems. *Journal of Differential Equations*, 2(1966), 293–304.

[3] M. Hanan. Oscillation criteria for third order linear differential equation. *Pacific J. Math.*, 11(1961), 919–944.

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CONJUGACY OF AN INVERTIBLE QUASILINEAR DIFFERENCE EQUATIONS[1]

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We consider the invertible quasilinear difference equations in Banach space:

$$x(t+1) = A(t)x(t) + f_1(t, x(t)) \tag{1}$$

and

$$x(t+1) = A(t)x(t) + f_2(t, x(t)) \tag{2}$$

where

$$|f_i(t, x) - f_i(t, x')| \leq \varepsilon(t)|x - x'|, \quad i = 1, 2,$$

$$\sup_x |f_1(t, x) - f_2(t, x)| \leq N(t) < +\infty.$$

Suppose that invertible linear equation

$$x(t+1) = A(t)x(t)$$

has Green type mapping $G(t, s)$.

THEOREM 1. *If*

$$\sup_{t\in\mathbb{Z}} \sum_{s=-\infty}^{+\infty} |G(t, s)|N(s) < +\infty,$$

$$\sup_{t\in\mathbb{Z}} \sum_{s=-\infty}^{+\infty} |G(t, s)|\varepsilon(s) = q < 1$$

then the difference equations (1) *and* (2) *are globally conjugated.*

REFERENCES

[1] Liangping Jiang. Ordinary dichotomy and global linearization. *Nonlinear Anal., Theory Methods Appl.*, **70** (7):2722-2730, 2009.

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CROSS STREAM PARTICLE MIGRATION IN FIBER SUSPENSION FLOWS[1]

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The presence of rigid or flexible fibers in a suspension increases its effective viscosity. In Dinh-Armstrong model the increase in the local stress tensor is expressed as $\eta N_p(\phi) D : a^{(4)}$, where D is the symmetric part of velocity gradient tensor and $a^{(4)}$ is fourth order orientation tensor, ϕ is the fiber volume fraction and $N_p(\phi)$ is the concentration dependent particle number [1].

Modelling and simulation of fiber orientation dynamics has attracted a lot of research and so has the estimation of the function $N_p(\phi)$. In most simulations used in industry the local volume fraction ϕ is assumed to be constant even though local variations in fiber volume fraction are observed. Recently the interest in including a model of the evolution of fiber volume fraction in industrial simulations has grown [1].

Few models for fiber cross stream migration are available, see [1] for fiber suspensions, [2] for macromolecules and [3] for spherical particles. Due to the dimensions of industrial fibers, brownian diffusion is negligible and hydrodynamical fiber-fiber [2] and fiber-wall [4] interactions prevail.

The fiber volume fraction ϕ satisfies the conservation law

$$\phi_t + (v + j) \cdot \nabla\phi = 0$$

where v denotes the bulk velocity of the suspension and j is a concentration flux quantifying the deviation of fiber velocity from the local suspension velocity. The flux j accounts for the cross stream migration. We review the approaches taken in [1], [2], taking a more general wall effect model [4]. Finally, the migration is included in the fiber orientation model (of Folgar-Tucker type) so that the fiber orientation tensor $a^{(2)}$ is scaled by ϕ.

REFERENCES

[1] P.J. Krochak, J.A Olson and D.M. Martinez. Near-wall estimates of the concentration and orientation distribution of a semi-dilute rigid fibre suspension in Poiseuille flow. *J. Fluid Mech.*, **653** 431–465, 2010.

[2] R.L. Schieck and E.S.G. Shaqfeh. A nonlocal theory for stress in bound, Brownian suspensions of slender, rigid fibres. *J. Fluid Mech.*, **296** 271-324, 1995.

[3] R.J. Phillips, R.C. Armstrong, R.A. Brown, A.L. Graham and J.R. Abbott. A constitutive equation for concentrated suspensions that accounts for shear-induced particle migration. *Phys. Fluids*, **4** (1), 30–40, 1992.

[4] A. Ozolins and U. Strautins. Simple models for wall effect in fiber suspension flows. *Math. Model. Anal.*, **19** (1), 75–84, 2014.

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MATHEMATICAL OLYMPIAD SYSTEM IN LATVIA - THEN AND NOW

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Every academic year A.Liepa's Correspondence Mathematics School organizes mathematical competitions and Olympiads [1] for students from Grade 4 to 12.

In the past few years some questions have raised:

- What is the purpose of these competitions?
- Are they necessary or not?
- How difficult are problems of these competitions?
- Are in many years established traditions still suitable for students or whether there is a need for changes?

Rather often opinions of teachers and students differ from the organizers opinion. A survey about advanced teaching of mathematics in Latvia was conducted to find out what teachers think about this topic. The survey results (answers of 250 teachers from different regions of Latvia) as well as students results in mathematical competitions will be considered in the report.

REFERENCES

[1] A.Liepa's Correspondence Mathematics School web site http://nms.lu.lv

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HOW QUASILINEARIZATION HELPS TO TREAT RESONANT PROBLEMS

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It is known that a nonresonant ($k \neq \pi n$, where n is an integer) problem

$$x'' + k^2 x = f(t, x, x'), \quad x(0) = 0, \; x(1) = 0, \tag{1}$$

has a solution if f is a continuous and bounded function.

This is not true for resonant problems.

However sometimes it is possible to modify a resonant problem so that the modified problem has a solution which at the same time solves the original problem.

We provide the example and explain how the quasilinearization works.

REFERENCES

[1] N. Sveikate. Resonant Problems by Quasilinearization. *Int. J. Diff. Equations*, **2014** , Article ID 564914, 2014. (to appear).

[2] I. Yermachenko and F. Sadyrbaev. Quasilinearization and multiple solutions of the Emden-Fowler type equation. *Mathematical Modelling and Analysis*, **10** (1), pp. 41-50, 2005.

[3] I. Yermachenko and F. Sadyrbaev. Types of solutions and multiplicity results for two-point nonlinear boundary value problems. *Nonlinear Analysis*, **63** , pp. e1725–e1735, 2005.

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M-VALUED BORNOLOGIES ON FAMILIES OF L-FUZZY SETS[1]

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In order to apply the conception of boundedness, so crucial in the theory of metric spaces, to the case of a general topological space Hu Sze-Tsen introduced the notion of a bornology (S.-T. Hu, *Boundedness in a topological space*, J. Math. Pures Appl., **78** (1949), 287–320.) Actually a bornology on a set X is an ideal $\mathcal{B}$ of its subsets containing all singletons. Given bornological spaces $(X, \mathcal{B}_X)$ and $(Y, \mathcal{B}_Y)$ a mapping $f : (X, \mathcal{B}_X) \to (Y, \mathcal{B}_Y)$ is called bounded if the image $f(A)$ of every set $A \in \mathcal{B}_X$ belongs to $\mathcal{B}_Y$.
Important examples of bornological spaces are: a topological space and its relatively compact sets; a metric space and its bounded subsets; a uniform space and its totally bounded subsets.
We introduce the concept of an M-valued L-fuzzy bornology, or an (L, M)-bornology for short, where $(L, \wedge_L, \vee_L)$ is a complete lattice and $(M, \wedge_M, \vee_M, *)$ as a cl-monoid (G. Birkhoff, *Lattice Theory*, AMS Providence, RI, 1995) whose underlying lattice is completely distributive. Namely, an (L, M)-bornology on a set X is a mapping $\mathcal{B} : L^X \to M$, such that:

1. (1) $\mathcal{B}(\{x\}) = 1_M \quad \forall\, x \in X$ (1_M is the top element of lattice M);

2. (2) $U \subseteq V \Longrightarrow \mathcal{B}(U) \geq \mathcal{B}(V)\ \forall U, V \in L^X$;

3. (3) $\mathcal{B}(U \cup V) \geq \mathcal{B}(U) * \mathcal{B}(V)\ \forall U, V \in L^X$.

Some results and problems concerning the category of (L, M)-bornological spaces will be discussed. (L, M)-bornologies generated by fuzzy metrics will be constructed and (L, M)-bornologies reflecting compactness-type properties in Chang-Goguen L-fuzzy topological spaces will be described.

In the special case when $M = \{0, 1\} (= 2)$ is a two-point lattice, $(L, 2)$-bornologies were considered in M. Abel, A.Šostak, *Towards the theory of L-bornological spaces*, Iranian J. Fuzzy Syst., **8** (2011) 19–28 and in the special case when $L = \{0, 1\} (= 2)$ is a two-point lattice $(2, M)$-bornologies were studied in I. Uļjane, A.Šostak, *Bornological structures in the context of L-fuzzy sets*, 8th Conf. European Soc. Fuzzy Logic and Technology, Atlantis Premium Proc., 481-488.

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ROBUST STATISTICAL INFERENCE USING GENERALIZED EMPIRICAL LIKELIHOOD FUNCTION

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Robust regression estimators are commonly used in case of the presence of outliers in datasets. The empirical likelihood method introduced by Owen [1] in 1988 is suitable and frequently used for robust statistical inference. The reason is simply its applicability for M-estimators as already shown in the pioneering paper by Owen [1].

To obtain efficient and robust linear regression estimators Bondell and Stefanski [2] proposed the two-stage generalized empirical likelihood method. Their idea is to combine the generalized empirical likelihood methods for regression problems with some robust variance estimation procedures. We analyze their method by simulations and propose to use it for related ANOVA models.

REFERENCES

[1] A. B. Owen. Empirical likelihood ratio confidence intervals for a single functional. *Biometrika*, **75** (2):237–249, 1988.

[2] H. D. Bondell and L. A. Stefanski. Efficient robust regression via two-stage generalized empirical likelihood. *Journal of the American Statistical Association*, **108** (502):644–655, 2013.

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SOLUTIONS OF BOUNDARY VALUE PROBLEMS FOR SOME NONLINEAR COUPLED SYSTEM

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The equation of simple pendulum given by $z'' = -a \sin z, \quad a > 0$ in a case of complex-valued function $z(t) = x(t) + i\, y(t)$ gives rise to a coupled nonlinear system

$$\begin{cases} x'' = & -a \sin x \cosh y, \\ y'' = & -a \cos x \sinh y. \end{cases} \tag{1}$$

Using the new variables $x = q_1$, $y = q_2$, $y' = p_1$, $x' = p_2$ the system (1) can be written as

$$\begin{cases} q_1' = & p_2, \\ q_2' = & p_1, \\ p_1' = & -a \cos q_1 \sinh q_2, \\ p_2' = & -a \sin q_1 \cosh q_2. \end{cases} \tag{2}$$

The system (2) is a system of canonical Hamiltonian equations with two degrees of freedom. The Hamiltonian function for (2) is given by

$$H(q_1,\, q_2,\, p_1,\, p_2) = p_1\, p_2 + a \sin q_1 \sinh q_2. \tag{3}$$

So trajectories of the system (1) lie on a surface determined by

$$H(x,\, y,\, y',\, x') = x'\, y' + a \sin x \sinh y = const. \tag{4}$$

Solutions of (1) with $H = 0$ are investigated. For these purposes different boundary conditions are considered, for instance

$$\begin{array}{l} x(0) = 0 = x(T), \\ y'(0) = 0 = y'(T); \end{array} \quad (5) \qquad \begin{array}{l} x(0) = \pi = x(T), \\ y'(0) = 0 = y'(T); \end{array} \quad (6) \qquad \begin{array}{l} x(0) = \pi, \;\; x(T) = 2\pi, \\ y'(0) = 0 = y'(T). \end{array} \quad (7)$$

REFERENCES

[1] S. S. Abdullaev. Construction of Mappings for Hamiltonian Systems and Their Applications. Lect. Notes Phys. 691 (Springer, 2006), ISBN-10 3-540-30915-2

[2] S. Bhalekar. Infinite-scroll attractor generated by complex pendulum model. Int. J. of Analysis, Hindawi, vol. 2013, article ID 368150http://dx.doi.org/10.1155/2013/368150