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NUMERICAL ANALYSIS FOR BOUNDARY-VALUE PROBLEMS OF VARIABLE COEFFICIENTS WITH PERIODICAL BOUNDARY CONDITIONS[1]

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In this talk, the finite difference scheme (FDS) for approximating periodic function's derivatives in a $2n+1$ point stencil is studied, obtaining higher order accuracy approximation. This method in the uniform grid with N mesh points is used to approximate the differential operator of the second and the first order derivatives in the space, using the multi-point stencil. The described methods are applicable for various mathematical physics problems involving periodic boundary conditions(PBC). In [1] the parabolic equations with constant coefficients are consider, here we analyse the boundary value-problems with variable coefficients. The solutions of some linear problems for parabolic type partial differential equations (PDE) with PBCs are obtained, using the method of lines (MOL) to approach the PDEs in the time and the discretization in space applying the FDS of different order of the approximation and finite difference scheme with exact spectrum (FDSES) [2].

The linear heat transfer equations with the variable coefficient can be written in the following form: $\boldsymbol{u_t(x,t) = k(x)(u(x,t))_{xx} + p(x)(u(x,t))_x + q(x)u(x,t) + f(x), u(x,0 = u_0(x),}$ where $k(x) > 0, p(x), q(x), u_0(x)$ are real functions, $x \in (0, L), t > 0$ are the space and time variables, L is the period, $u = u(x,t)$ is the unknown function (for ODE we have boundary value problem with $u = u(x)$).

We have the discrete equations ($x_j = jh, Nh = L, j = \overline{1,N}$) as a system of ODEs in following form: $\boldsymbol{\dot{U} = K(A_{2,n}(U) + P(A_{1,n}(U) + Q(U) + F, U(0) = U_0,}$ where $A_{1,n}, A_{2,n}$ are N-th order circulant matrices,$U = U(t)$, $F, \dot{U}, U_0$ are the column-vectors of the N order with elements $u_j(t), f(x_j(t)), u_t(x_j,t), u_0(x_j)$ K, P, Q are N-th order diagonal matrices with elements $k(x_j), p(x_j), q(x_j)$ (in the case of the constant matrices k, p, q we have Kronecker tensor products $k \bigotimes A_{2,n}, p \bigotimes A_{1,n}, q \bigotimes I$, I is N-order unit matrix).

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ALGORITHMS FOR NUMERICAL SOLVING OF SPECIAL TYPE EQUATIONS

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The equation

$$f(x) := \frac{a_1}{x^{b_1}} + \frac{a_2}{x^{b_2}} + \ldots + \frac{a_n}{x^{b_n}} = C,$$

where $n \geq 2$, $C > 0$, $0 < b_1 < b_2 < \ldots < b_n$, and $a_1, a_2, \ \ldots, a_n > 0$, appearing in some banks' calculations of annual percentage rate is considered. The attention is paid to the problem of finding the unique real positive root of this equation with high accuracy using as few as possible number of mathematical operations.

The proposed fast root-finding algorithm is the following iterative method

$$x_1 = \left(\frac{a_1 + a_2 + \ldots + a_n}{C}\right)^{n/(b_1+b_2+\ldots+b_n)}, \ x_{k+1} = x_k \cdot \left(\frac{f(x_k)}{C}\right)^{-\frac{1}{x_k}\cdot\frac{f(x_k)}{f'(x_k)}}, \ k \geq 1.$$

Practical testing shows that a few iterations are enough (so x_4 is very close to the unique positive root). A similar algorithm that does not contain $f'(x)$ is also considered.

REFERENCES

[1] http://en.wikipedia.org/wiki/Annual_percentage_rate

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THE NUANCES OF ALGORITHMS DEVELOPMENT

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The development of algorithms is an integral part of mathematical modelling and numerical methods implementation. An algorithm is a system of clearly defined rules that determine the transformation of input information into output information. In literature it is mentioned that algorithm has such obligatory properties, ie. the algorithm should be:

- discreet, ie. the process of information transformation should be divided in simple steps;
- unambiguous, ie. the rules and sequence of steps should have only one meaning or interpretation and leading to only one result;
- fruitful, ie. the expected result should be reached executing finite number of steps;
- mass, ie. devised to a solution of definite class of problems.

However this list of properties should be supplemented for learning purposes, as well as for development purposes of algoritms or computer programms intended for use in a wider range of applications. The algorithm should be complete and informative too. The complete algorithm is an algorithm which provides all possible execution paths in the context of definite problem's solution. Incomplete algorithm has shortcommings and as a result, in some cases the problem remains unresolved. Informative algorithm informs user of the outcome even, if incorrect data is entered or when solution does not exist and in other cases aimed for the user to understand the error and be able to make decision on the further problem solution, for example, to enter different input data or choose another method. Especially it is useful and voluable for learning process, for example, mastering and implementing numerical methods.

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APPLICATION OF RADIO-ASTRONOMICAL DATA PROCESSING METHODS FOR RADAR-VLBI OBSERVATION OF NEAR-EARTH CELESTIAL AND ARTIFICIAL SPACE BODIES

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The VLBI (Very Long Baseline Interferometry) method is well known radio astronomical technique. Nowadays VLBI elements have been successfully applied to trajectory measurements of artificial Earth satellites, Solar system bodies, space debris and interplanetary space station. Time-consuming numerical data processing in the case of VLBI observations is mandatory.

The VLBI data processing is performed in at least two major steps. The first, correlation step with high computational complexity acts on the raw sampled signals from the pairs of VLBI stations, yielding signal interference functions, so called "correlation function spectra" and basic parameters (delays) needed for correlation function construction. Specific methods for time and frequency delay compensations are reviewed for Near-Earth objects and additional time-frequency analysis are applied to the data for fractional step compensations in the correlation integral constructions. High sampling rates allows high sensitivity and better random noise elimination, altogether leading to substantial numerical complexity of the raw data processing.

The correlation function spectra with the corresponding delays are used in the second step of data processing, which ultimately leads to the measured orbital parameters of observed object, such as angular positions, velocities, etc.

Data of several observations of space debris (non-functional GPS and Glonass satellites), ionosphere and celestial bodies (asteroid DA14 2012), are processed and results are presented and discussed.