Quantum rejection sampling

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Resampling

Classical $\boldsymbol{p} \to \boldsymbol{s}$ resampling problem

- **Given:** $\boldsymbol{p}, \boldsymbol{s} \in \mathbb{R}^n_+$ with $\|\boldsymbol{p}\|_1 = \|\boldsymbol{s}\|_1 = 1$
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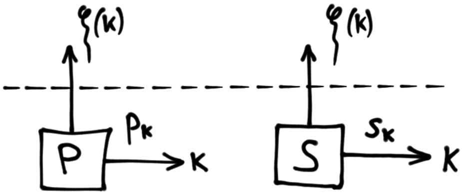
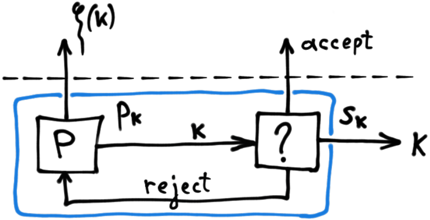
- **Note:** Samples are pairs $(k, \xi(k))$ where $\xi(k)$ is not accessible



Classical rejection sampling

Algorithm



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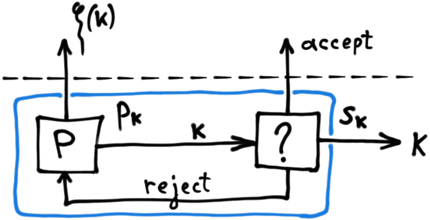

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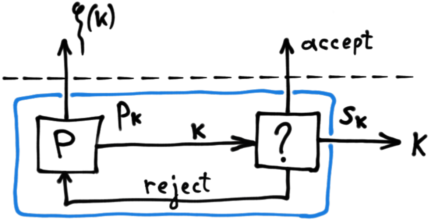

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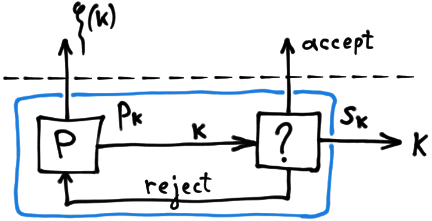

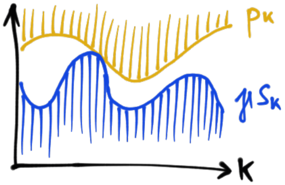

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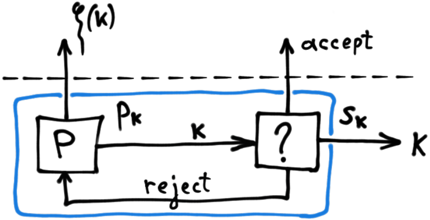

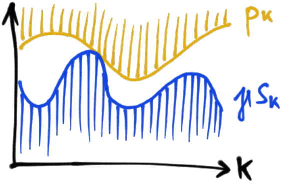

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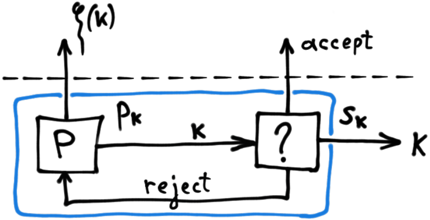

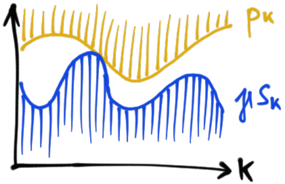

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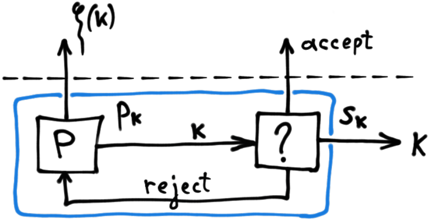

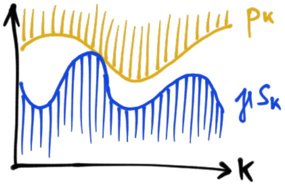

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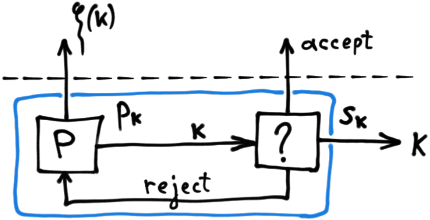

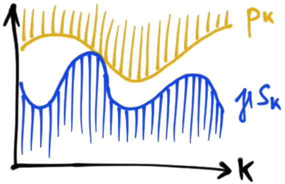

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- Introduced by von Neumann in 1951
- Has numerous applications:
  - Metropolis algorithm [MRRTT53]
  - Monte-Carlo simulations
  - optimization (simulated annealing), etc.

Quantum computing in 60 seconds

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Quantum states

$$|\psi\rangle = \begin{pmatrix} \psi_1 \\ \vdots \\ \psi_n \end{pmatrix} \in \mathbb{C}^n$$

$$\|\psi\|_2 = \sum_{i=1}^n |\psi_i|^2 = 1$$

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$$|\alpha\rangle|\beta\rangle = \begin{pmatrix} \alpha_0 \\ \alpha_1 \end{pmatrix} \otimes \begin{pmatrix} \beta_0 \\ \beta_1 \end{pmatrix} = \begin{pmatrix} \alpha_0 \begin{pmatrix} \beta_0 \\ \beta_1 \end{pmatrix} \\ \alpha_1 \begin{pmatrix} \beta_0 \\ \beta_1 \end{pmatrix} \end{pmatrix} = \begin{pmatrix} \alpha_0\beta_0 \\ \alpha_0\beta_1 \\ \alpha_1\beta_0 \\ \alpha_1\beta_1 \end{pmatrix}$$

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(Partial) measurement

$$\begin{pmatrix}\boldsymbol{\psi}_0 \\ \boldsymbol{\psi}_1\end{pmatrix} \Rightarrow \begin{cases}\boldsymbol{\psi}_0/\|\boldsymbol{\psi}_0\|_2 \text{ w.p. } \|\boldsymbol{\psi}_0\|_2^2 \\ \boldsymbol{\psi}_1/\|\boldsymbol{\psi}_1\|_2 \text{ w.p. } \|\boldsymbol{\psi}_1\|_2^2\end{cases}$$

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3. Measure the first register:
   - w.p. $\|\boldsymbol{\delta}\|_2^2$ the state collapses to

$$\sum_{k=1}^{n} \hat{\delta}_k |k\rangle|\xi(k)\rangle$$

   where $\hat{\delta}_k = \delta_k / \|\boldsymbol{\delta}\|_2$

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Subroutine

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Goal: preparing $|\sigma\rangle$

- What $\boldsymbol{\delta}$ should we choose?
- We are done if $\boldsymbol{\sigma} \cdot \hat{\boldsymbol{\delta}} \geq \sqrt{1-\varepsilon}$ where $\hat{\boldsymbol{\delta}} = \boldsymbol{\delta}/\|\boldsymbol{\delta}\|_2$

Optimization

Problem

- $\min_{\boldsymbol{\delta}} 1/\|\boldsymbol{\delta}\|_2$ s.t. $\boldsymbol{\sigma} \cdot \hat{\boldsymbol{\delta}} \geq \sqrt{1-\varepsilon}$

Optimization

$$\sum_{k=1}^{n} \left(\sqrt{|\pi_k|^2 - |\delta_k|^2}\,|0\rangle + \delta_k\,|1\rangle\right)|k\rangle|\xi(k)\rangle$$

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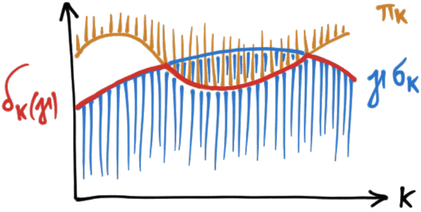
Optimal solution

- Let $\delta_k(\gamma) = \min\{\pi_k, \gamma\sigma_k\}$



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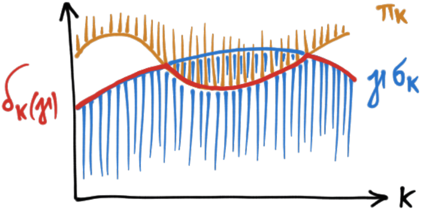

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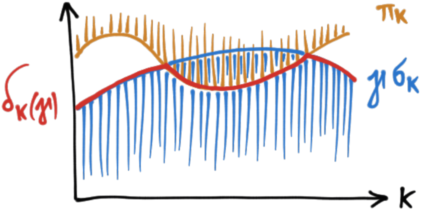

Main theorem

The quantum query complexity of the ε-approximate $\boldsymbol{\pi} \to \boldsymbol{\sigma}$ quantum resampling problem is $\Theta(1/\|\boldsymbol{\delta}(\bar{\gamma})\|_2)$

Applications

Implicit use

- Synthesis of quantum states [Grover, 2000]
- Linear systems of equations [Harrow, Hassidim and Lloyd 2009]
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Future applications

- Preparing PEPS [Schwarz, Temme, Verstraete, 2011]
- More...

Thank you!

Funding: