10th International Conference
APLIMAT 2011
Faculty of Mechanical Engineering - Slovak University of Technology in Bratislava
Section:

On comparison of univariate forecasting methods: the case of Latvian residential property prices

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Abstract. This paper investigates the forecasting ability of different univariate forecasting techniques (local regressions, unobserved component model), compared with the standard ARIMA approach. A forecasting exercise is carried out with each method, using monthly price time series on residential property prices in Latvia. The accuracy of the different methods is assessed by comparing several measures of forecasting performance based on the out-of-sample predictions for various horizons.

Key words and phrases. Unobserved components model; ARIMA models; Forecasting comparison; local linear regression.

Mathematics Subject Classification. Primary 91B84, 62M10; Secondary 62G08.

1 Introduction

The presence of a turning point at the end of a sample introduces a degree of uncertainty in econometric forecasting. Deterministic trend-based forecasting strategies are not relevant in this case. Cointegration relationship also may seem to be broken thus not providing a solid basis for making projections of future paths of economic variables. Uncertainty about the future behavior of exogenous variables due to turn in economic cycle makes it difficult to use traditional multivariate forecasting methods. In this case univariate forecasting techniques may serve as an additional work aid. The purpose of the article is to investigate the forecasting performance of several univariate modelling methods (unobserved component model, local regressions and traditional ARIMA models) applied to Latvian residential property prices.

The development of real estate prices serves as an important economic indicator, closely connected to economic and credit cycles. As the behavior of the property prices influences the performance of the whole financial system, the forecasting of its future developments is an important task. Over the last decade Latvian residential property prices experienced dramatic changes. Joining the European Union in 2004 provided access to cheap credits. The economic boom that followed the EU accession led to growing wages and therefore the demand for real estate and affordability of the loans for house purchase increased ([1]). Credit and property prices grew in mutually reinforcing manner, producing a speculative real estate price bubble, which burst in April 2007. The price drop on the real estate market coincided with a severe downturn in economic activity in Latvia, which was reinforced by the world-wide financial and economic crises. During 2007 - 2009 Latvian residential property prices fell by more than 70%. After hitting the bottom in the summer of 2009, the prices exhibited moderate growth. The idea of the exercise appeared at the end of 2009, when future prospects of the developments of the property prices were highly uncertain and the deep recession made the forecasting of the fundamental determinants of property prices comlicated for relatively long time horizons. Different classes of univariate models were fitted to residential property price time series on the estimation sample from January 1999 to December 2009 and 12 months ahead forecasts were produced. Now, when 10 of 12 actual values of property prices time series for 2010 are already known, the precision of forecasts could be accessed.

The paper is organised as follows: Section 2 describes the theoretical UC model based on simple Integrated Random Walk model for the trend and results, obtained employing this model, including some modifications. Section 3 and Section 4 describe the nonparametric regression methods and benchmark ARIMA class models, respectively. Section 5 analyses the predictive performance of our models for Latvian residential property prices. Section 6 concludes.

2 Forecasts based on structural time series model

One of the popular ways of modelling time series is a structural time series model, which is set in terms of components having a direct economic interpretation. In the most common form additive structural model has the folowing form

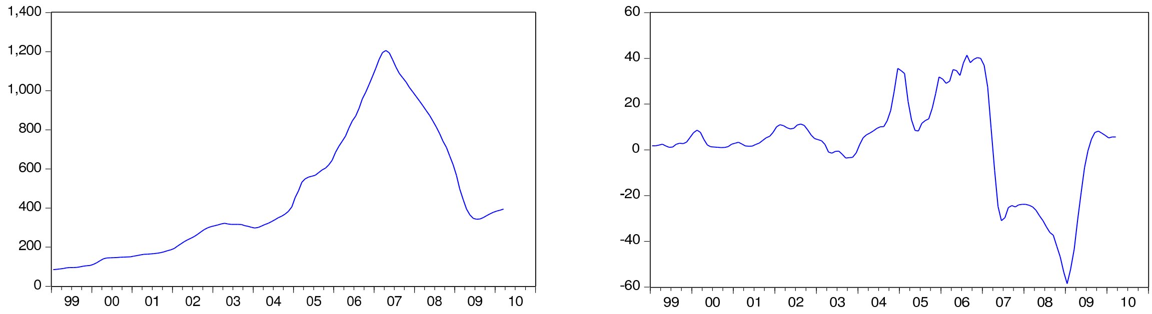
$$y_t = T_t + C_t + S_t + \epsilon_t, \quad t = 1, \ldots, T, \tag{1}$$

where T_t - trend, C_t - cyclical, S_t - seasonal, ϵ_t - irregular component. Typically the irregular component is assumed to be white noise with zero mean and variance σ_ϵ^2. In some cases (like in [3]) the cyclical component is dropped and cyclical movements are incorporated into trend component. The new framework elaborated for structural models by Harvey (see e.g. [4], [5]) made the models more flexible, in particular, by letting the level and slope parameters of trend to change over time. Harvey and Jaeger in [5] proposed to fit trend as

$$T_t = T_{t-1} + D_{t-1} + \zeta_t, \quad \zeta_t \sim NID(0, \sigma_\zeta^2), \tag{2}$$

$$D_t = D_{t-1} + \eta_t, \quad \eta_t \sim NID(0, \sigma_\eta^2), \tag{3}$$

where D_t is the slope and the normal white noise disturbances ζ and η are independent of each other. In case $\sigma_\eta^2 = 0$, the formula (2) reduces to random walk with drift and if, in addition, $\sigma_\zeta^2 = 0$, it reduces to deterministic linear time trend; the case when $\sigma_\zeta^2 = 0$ and $\sigma_\eta^2 \neq 0$ corresponds to integrated random walk (IRW). So, the proposed way of modelling time trend allows for time-varying parameters and incorporates possibilities of random walk and linear trend as limiting cases. The key idea of Harvey (see [4]) was to handle structural models in the state space form with the state of the system representing the various unobserved components such as trends and cycles. The forecast in this type of structural time series model are constructed automatically by the Kalman filter. The trend and other unobservables



(a) (b)

Figure 1: Estimated trend (a) and trend derivative (b) for Latvian residential property prices during January 1999 - October 2010.

are extracted by a smoothing algorithm. The parameters, which govern the evolution of the observed series, are estimated by maximum likelihood, again using the Kalman filter. Thus the whole model is handled within a unified statistical framework, which produces optimal estimates with well defined properties.

3 Empirical results

As Latvian residential property prices time series exhibits no seasonal variation, the seasonality term in equation (1) was omitted and a cycle was incorporated into the trend-cycle component (2) - (3), for which integrated random walk specification was chosen. IRW model is known to be particularly useful for describing large smooth changes in the trend ([2]).

In classical Kalman filter framework our model has the following representation: the state equation

$$\xi_{t+1} = F\xi_t + \nu_{t+1} \tag{4}$$

and the observation equation

$$y_t = H'\xi_t + \epsilon_t,$$

where

$$\xi_t = \begin{pmatrix} T_t \\ D_t \end{pmatrix}, \; F = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, \; H = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \nu_t = \begin{pmatrix} 0 \\ \eta_t \end{pmatrix}.$$

The model appears to be very simple and parsimonious with the only unknown parameter - σ_η^2. Following Garcia-Ferrer and Queralt [2], we will call the slope component D_t a trend derivative.

Estimation of the parameters and extracting of the components was carried out by the means of econometric package EViews7. The estimated trend and the trend derivative are shown on Figure 1.

Garcia-Ferre and Queralt (see [2]) argues, that the trend derivative can be used as a device for anticipating peaks and troughs, in particular, when the derivative reaches its maximum value, the recession is to be expected, and the recession is confirmed, when the derivative becomes negative. Indeed, also in our case the peaks in derivative precedes the turning points in the levels of time series for some 2-3 months and thus can serve to improve quantitative forecasts in the vicinity of turning points.

The trend prediction in the IRW trend model is a straight line with a constant slope equal to the last value of the derivative. This seems to be a rather restrictive and conservative assumption given the evolution of the derivative through time. Two alternatives seem to be open: (1) propose different (more flexible) trend model, like Smooth Random Walk (SRW) or Double Integrated Autoregressive model (DIAR); and (2) direct modelling of IRW trend derivative and obtaining forecasts from its univariate model, as proposed in [2]. We have left the first alternative for further research and followed Garcia-Ferre identifying and estimating corresponding ARIMA models for the trend derivative. The two alternatives, suggested by the inspection of ACF, PACF and the analysis of unit root tests are AR(3) model with one root close to unity ("quasi-stationary") and non-stationary ARI(2,1) model. Therefore both competing models are rather close. Despite this fact, the forecasts, produced by the above models are rather different (see Figure 2 and Tables 2 and 3). In the following, we will call both model "modified structural models" and refer in tables as UC_AR(3) and UC_ARI(2,1) (unobserved component model with trend derivative forecasted by AR(3) and ARI(2,1) models correspondingly).

4 Forecasts based on the local linear smoothing method

For the time series observations $y_1, y_2, \ldots, y_T$ consider the commonly used kernel regression estimator introduced by Nadaraya [6] and Watson [7] defined by

$$\hat{T}_{t_0} = \frac{\sum_{t=1}^{T} y_t K(\frac{t-t_0}{h})}{\sum_{i=1}^{T} K(\frac{t-t_0}{h})},$$

where T_{t_0} denotes the trend function at a fixed timepoint t_0, K is a kernel function and h - the bandwidth parameter. It is well known that this method has a big drawback - it has the so called design bias (see, for example, [8] or [9]). This issue is more pronounced at the boundary regions, therefore this method is not quite suitable for the forecasting purposes.

The local linear smoothing is another nonparametric regression method, which minimizes the following expression

$$\sum_{i=1}^{T} \{y_i - a - b(i-t)\}^2 K_h(i-t),$$

with respect to a and b. Denote by $\hat{a}_t$ and $\hat{b}_t$ the least-squares solutions, where the subscript t is used to indicate that the solution depends on the given timepoint t. Then T_t is estimated by the local intercept $\hat{a}_t$, which admits the explicit expression

$$\hat{T}_t = \hat{a} = \frac{\sum_{i=1}^{T} w_{t,i} y_i}{\sum_{i=1}^{T} w_{t,i}},$$

where

$$w_{t,i} = K_h(i-t)\{S_{T,2}(t) - (i-t)S_{T,i}(t)\}$$

and

$$S_{T,j} = \sum_{i=1}^{T} K_h(i-t)(i-t)^j.$$

It can be shown that using the local linear smoothing method the design bias vanishes, thus it improves over the usual Nadaraya-Watson kernel regression estimator. It is also possible to consider more general local polynomial fitting methods described by [9] in details. However, practically the local linear smoothers are used most commonly.

We have implemented the nonparametric regression smoothers in program **R**. It is well known that the kernel choice is not essential, thus we choose the standard Gaussian kernel. However, the smoothing parameter or the bandwidth choice is crucial. There exist many methods for the bandwidth selection which roughly can be divided into cross-validation and plug-in methods. Moreover, for the time-domain smoothing due to the local dependence the bandwidth selectors for independent samples do not work well (see, for example, Section 6 in [8]). Therefore for the comparison we examined several built-in automatic methods for bandwidth selection such as 1) cross-validation (`sm.regression` command choosing the method `cv`); 2) plug-in method (command `dpill`) and 3) iterative method based on autoregressive regresion errors (command `sm.regression.autocor`). In all cases we obtain similar results, that is, $h = \{2.39; 1.23; 1.14\}$, respectively. The reason may be very simple: the time series data are already quite smooth. Therefore we present here (see Section 6) only the forecasts using the plug-in method (command `dpill` which works under the package `KernSmooth`).

5 Forecasting based on ARIMA models

It has become common to use the best-fitting ARIMA class model (see [10]) as a benchmark, investigating the performance of different class models. The graph of Latvian residential property prices time series and the correlogram of levels are suggestive of non-stationary process, which was supported by the results of unit root tests (ADF and KPSS tests results both confirm I(1) process). That's why we were looking for the best model among the class of Integrated models of first order (ARMA models for first differences). ACF and PACF properties of differenced series were used to select the orders of autoregressive and moving average polinomials of the models, which were then estimated by maximum likelihood. Taking into account Akaike (AIC) and Schwartz Bayesian (BIC) information criteria, the best models from this class on the considered observation sample from January 1999 to December 2009 appeared to be ARI(2,1) and ARIMA(1,1,1) (both fitted to logs of dependant variable) with the impulse dummies for periods 2005M1 and 2009M2. Estimated equations have the following form:

$$dln(y_t) = 0.14d2005m1 - 0.11d2009m2 + 0.37(dln(y_{t-1}) + dln(y_{t-2})), \tag{5}$$

$$dln(y_t) = 0.12d2005m1 - 0.08d2009m2 + 0.87dln(y_{t-1}) - 0.37\hat{\epsilon}_{t-1}. \tag{6}$$

All estimated parameters are significant on 1% significance level. Both dummies serve to fix outliers in residuals. Positive shock in January of 2005 is connected with the change of the peg of Latvian currency - Lats from SDR currency basket to euro. Prior to 2005 property prices were denominated in USD, and the change to euro was used by property owners to rise the prices. The negative shock in February of 2009 was caused by the negative trends in real economy, which accelerated fall in property prices.

According to the both information criteria, ARI(2,1) is slightly superior to ARIMA(1,1,1) due to more parsimonious specification (see Table 1).The residual tests (for autocorrelation, normality and heteroscedasticity, not shown to save the space) revealed good statistical properties of both models ((5) and (6)).

Fitted ARIMA models (5) and (6) were employed to forecast the values of property prices series for a year ahead (January to December 2010). The forecasting results are shown and discussed in the next section.

Table 1: The values of Information criteria for the ARI(2,1) and ARIMA(1,1,1) models.

Model	ARI(2,1)	ARIMA(1,1,1)
AIC	-4.48	-4.38
SBC	-4.40	-4.29

6 Forecasting results

To compare the forecasting performance of different models, all the models were estimated, using data only from estimation set (from January 1999 to December 2009). Then the estimated models were used to predict next 12 values (from January 2010 to December 2010). Figure 2 shows forecasts produced by the models and actual values of data series, which are already known (from January 2010 to October 2010).

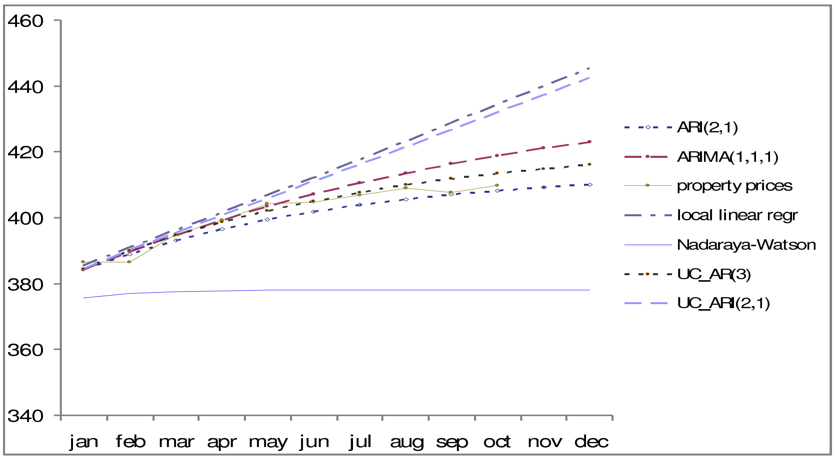


Figure 2: Different forecasts based on previously described methods together with the actual values of the data series.

To access accuracy of forecasts, we used two measures MSE and MAPE, based on prediction errors ϵ_t, that is

$$e_t = y_t - \hat{y}_t,$$

where y_t - the observed value from the test set and $\hat{y}_t$ - the forecast for the time moment t, based on the values from estimation set. The **predictive mean squared error (MSE)** uses squared residuals,

$$MSE = \frac{\sum_{t=1}^{n} e_t^2}{n},$$

where n is a number of forecasts. The **mean absolute percentage error (MAPE)** considers the relative absolute error of each forecast,

$$MAPE = \frac{\sum_{t=1}^{n} |\frac{e_t}{y_t}|}{n}.$$

Tables 2 and 3 show the calculated measures for different models.

Tables 2 and 3 show that the overall modified structural model with AR(3) process for trend derivative performs the best according to the both criteria. The forecasts from ARIMA

Table 2: Forecast MSE for horizons 1-10.

period	lin.local regr	Kernel regr.	ARI(2,1)	ARIMA(1,1,1)	UC_AR(3)	UC_ARI(2,1)
2010M01	**1.25**	116.41	4.57	5.50	4.34	4.34
2010M02	20.85	91.93	**6.11**	10.18	11.54	13.03
2010M03	1.98	304.22	3.08	**0.05**	0.11	0.26
2010M04	5.74	456.86	6.69	**0.02**	0.25	2.18
2010M05	7.59	683.90	21.12	**0.44**	3.78	2.93
2010M06	55.21	717.28	8.35	5.59	**0.11**	38.22
2010M07	116.31	832.54	8.81	13.17	**0.62**	86.08
2010M08	201.62	957.49	11.45	20.85	**0.79**	153.71
2010M09	448.05	871.90	**0.32**	76.01	17.59	362.82
2010M10	607.41	1000.55	**2.21**	82.55	13.86	492.44
MSE	146.60	603.31	7.27	21.44	**5.30**	115.60

Table 3: Forecast MAPE (in %) for horizons 1-10.

period	lin.local regr	Kernel regr.	ARI(2,1)	ARIMA(1,1,1)	UC_AR(3)	UC_ARI(2,1)
2010M01	**0.29**	2.79	0.55	0.61	0.54	0.54
2010M02	1.18	2.48	**0.64**	0.83	0.88	0.93
2010M03	0.36	4.42	0.44	**0.05**	0.08	0.13
2010M04	0.60	5.35	0.65	**0.03**	0.12	0.37
2010M05	0.68	6.47	1.14	**0.16**	0.48	0.42
2010M06	1.84	6.62	0.71	0.58	**0.08**	1.53
2010M07	2.65	7.09	0.73	0.89	**0.19**	2.28
2010M08	3.47	7.57	0.83	1.12	**0.22**	3.03
2010M09	5.19	7.24	**0.14**	2.14	1.03	4.67
2010M10	6.02	7.72	**0.36**	2.22	0.91	5.42
MAPE	2.23	5.77	0.62	0.86	**0.45**	1.93

models also are fairly close to actual data; ARI(2) seems to be the second best. The linear local regression smoother captured well the slope for the beginning of the test sample: forecasts for the first 5 months are accurate, but the deviation is rather big for the rest of the test sample. Surprisingly, but the forecasts from the modified structural model with ARI(2,1) differ very much from the first structural model, despite the fact that actually the two models for the slope are rather close. However, those results show the improvement compared to the structural model with a pure random walk slope (not shown due to big deviations from actual series and other forecasts: forecasted value for October 2010 was 475, much worse than local linear regression). Nadaraya-Watson method also was not a success. So, forecasting fairly smooth time series after the turning point, usage of the local linear regression can be recommended only for short time horizons. ARIMA models performed fairly well for all horizons (1-10) with MAPE not exceeding 0.62%. The performance of the modified structural model crucially depends on the choice of the model for forecasting trend derivative.

7 Conclusions

In this paper we investigated the forecasting ability of different univariate forecasting techniques (local regressions, modified structural model, standard ARIMA approach). A forecasting example was carried out with each method, using monthly price time series on residential property prices in Latvia, which has recently experienced turn in the trend. The accuracy of the different methods was assessed by comparing the forecasts MSE and MAPE based on the out-of-sample predictions for 10 horizons. The modified structural model with AR(3) process for trend derivative performed the best according to the both criteria, with MAPE less than 0.5%. ARIMA models performed fairly well for all horizons (1-10) with MAPE not exceeding 0.62% and the local linear regression gave accurate forecasts for short horizons (up to 5). Nadaraya-Watson method didn't look suitable for this example. But it should be mentioned that the results from the modified structural model should be interpreted with caution, as another specification for trend derivative yielded much worse results. This feature of the model calls for the further investigation. In particular, more flexible trend characterisations (like smoothed random walk or dependences of orders higher than one) seems an important area for future research.

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