# Distance-to-Default

## (According to KMV model)


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# Goal:

Calculation of Distance-to-Default according to KMV model (Kealhofer Merton Vasicek model)

**The aim:** European companies, both non-/defaulted, both non-/financial

## Contents:

- Theoretical background
  - KMV model
- Application using real data
  - Computation of Distance-to-Default
  - Computation of probability of default

# What is default?

Default happens when company has not paid debts.

Bankruptcy is a legal term - inability to pay own debts.

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Bankruptcy is a legal term - inability to pay own debts.

Default risk is the uncertainty surrounding a firm's ability to service debts and obligations.

We need to "measure" it somehow...

# KMV model

## Idea:

Firm's equity can be seen as a call option on the underlying asset.

Because at the maturity of debt bondholders receive their debts, equity holders take the rest.

Use:

- observable value and volatility of equity ($V_E$ and $\sigma_E$),
- unobservable value and volatility of firm's asset ($V_A$ and $\sigma_A$).

- Based on Black-Scholes option pricing theory.
- Equity is a call option on the value of assets of the company ($V_A$ considered as $C$, $V_E$ as $S$).
- Debt (D) is taken as a strike price ($D$ considered as $K$).

# KMV model

## Assumptions:

- Debt:
  homogeneous with time of maturity $T$
- Capital structure:
  $V_A(t) = D(t) + V_E(t)$
- Market perfection:
  ignore coupons and dividends, no penalty to short sales, ...
- Dynamic of the asset:
  assets are traded and follow geometric Brownian motion

$$dV_A = \mu_A V_A dt + \sigma_A V_A dW.$$

$V_A$ is value of the asset, $\sigma_A$ its volatility, $\mu_A$ drift and $dW$ is a Wiener process.

# KMV model

Due to Black-Scholes option pricing theory analogically to

$$C(t) = S(t) \cdot \Phi(d_1) - e^{-r(T-t)} \cdot K \cdot \Phi(d_2)$$

value of equaty can be priced as

$$V_E(t) = V_A(t) \cdot \Phi(d_1) - e^{-r(T-t)} \cdot D \cdot \Phi(d_2) \tag{1}$$

Using Ito's formula one can show

$$\sigma_E = \frac{V_A}{V_E} \cdot \frac{\partial V_E}{\partial V_A} \cdot \sigma_A \tag{2}$$

$V_A$ ($V_E$) - value of the asset (equity)

$\sigma_A$ ($\sigma_E$) - volatility of asset (equity)

$r$ - risk-free rate

$$d_1 = \frac{\log\left(\frac{V_A(t)}{D}\right) + (r - \frac{1}{2}\sigma_A^2)(T-t)}{\sigma_A\sqrt{T-t}}$$

$$d_2 = d_1 - \sigma_A\sqrt{T-t}$$

$T$ - time of debt's maturity

# KMV model - nonlinear system of equations

Thus, to find unobservable value and volatility of the asset one should solve the nonlinear system of equations:

$$\begin{cases} f_1(V_E, \sigma_E) = V_A(t) \cdot \Phi(d_1) - e^{-r(T-t)} \cdot D \cdot \Phi(d_2) - V_E(t) = 0 \\ f_2(V_E, \sigma_E) = \frac{V_A}{V_E} \cdot \Phi(d_1) \cdot \sigma_A - \sigma_E = 0 \end{cases}$$

The solution is unique as

$\frac{\partial f_1}{\partial V_A} = \Phi(d_1)$ (analogically to $\delta$ in Black-Scholes)

$f_1$ is increasing function of $V_A \Rightarrow f_1(V_A)$ has a unique solution. Analogically, $f_2(\sigma_E)$ has unique solution as well.

# KMV model - Distance-to-Default

Default happens when the value of company's asset falls below "default point" (value of the debt).

## Distance-to-Default

- distance between the expected value of the asset and the default point
- after substitution into a normal c.d.f one gets probability of default

$$DD(t) = \frac{\log(\frac{V_A}{D}) + (r - \frac{1}{2}\sigma_A^2)(T - t)}{\sigma_A \sqrt{T - t}}$$

And probability of default:

$$\mathrm{PD}(t) = \mathrm{P}\left[V_A \leq D\right] = \cdots = \Phi(-DD)$$

# KMV model - Distance-to-Capital

## Distance-to-Capital

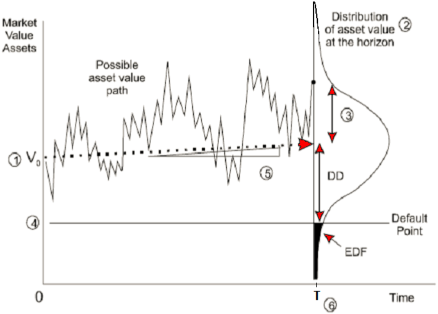
- processed from Distance-to-Default
- reason: DD does not include complexities related to financial firms
- according to [Larsen&Mange,2008] computed as:

$$DC(t) = \frac{\log(\frac{V_A}{\lambda \cdot D}) + (r - \frac{1}{2}\sigma_A^2)(T-t)}{\sigma_A \sqrt{T-t}}$$

where

$$\lambda = \frac{1}{1 - PCAR}$$

- PCAR capital requirement
(According to Basel Capital Accord I set to 8%)
- for DD we take $\lambda = 1$




1. Current asset value, $V_A(t)$
2. Distribution of the asset value at time $T$
3. Volatility of the future asset value at time $T$
4. Level of default point, $D$
5. Expected rate of growth in the asset value over the horizon
6. Length of the horizon, $T$

Source: [Crosbie&Bohn,2004]

# Needed data:

1. Risk-free interest rate
   Euribor
2. Price and number of stocks - weekly in Jan.2005 - Dec.2010
   - (almost) defaulted
     - financial: Commerzbank
     - nonfinancial: Arcandor
   - nondefaulted
     - financial: Credit-Suisse
     - nonfinancial: Volvo
3. Balance sheets (short- and long-term debts)

# Real data:

## Commerzbank

- 2nd biggest bank in Germany
- Aug.2008 announced acquisition of Dresdner bank
- Jan. 2009 help of 10 bil.eur from SoFFin (Fin.Market Stabil.Fund)

## Arcandor AG.

- German holding company
- May 2009 asked for government financial assistance
- 6th Jun 2009 announced inability to pay rents for stores
- 9th Jun 2009 bankruptcy

# Real data:

## Credit-Suisse

- swiss international financial company
- 2009 Bank of the Year by the International Financing Review

## Volvo

- Swedish producer of cars,trucks,..
- rapid growth in last years, 2007 bought Nissan

# Calculation:

First derive parameters:

1. Returns and volatility of equity using historical data (1 year)
2. Market value of equity = no. of stocks $*$ stock price
3. Risk-free interest rate Euribor
4. Time liabilities will mature in 1 year
5. Liabilities shot-term + one half of long-term

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First derive parameters:

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Then:

1. Simultaneously solve two nonlinear equations (in R) $\hookrightarrow$ get value and volatility of the asset
2. Calculate Distance-to-Default and probability to default

# Choosing the method

Example:

$V_E = 4740291$, $\sigma_E = 0.02396919$, $D = 33404048$, $r = 2.32$, $T - t = 1$, $V_A$-?, $\sigma_A$-?

- Starting value $(V_A = 4740291, \sigma_A = 0.02396919)$
  - Newton's $(8023027, 0.01416185)$, 3 iterations
  - Broyden $(8023027, 0.01416185)$, 3 iterations
  - Iterations $(8023027, 0.01416185)$, 6 iterations
  - One-dimensional $(8023027, 0.01416185)$, 7 iterations
- Starting value $(V_A = 0, \sigma_A = 0)$
  - Newton's $(4740291, 113620.9)$, Jacobian is singular
  - Broyden $(4740291, 113620.9)$, Jacobian is singular
  - Iterations $(8023027, 0.01416185)$, 6 iterations
  - One-dimensional $(8023027, 0.01416185)$, 7 iterations

# Newton's, Broyden

```
> fnewton <- function(x) {
+ y <- numeric(2)
+ d1 = (log(x[1]/Z) + (r + x[2]^2/2)*T)/x[2]/sqrt(T)
+ d2 = d1 - x[2]*sqrt(T)
+ y[1] <- S0 - (x[1]*pnorm(d1) - exp(-R*T)*D*pnorm(
  d2))
+ y[2] <- sigmaS * S0 - pnorm(d1)*x[2]*x[1]
+ y
+ }
> nleqslv(c(VE,SE), fnewton, control=list(btol=.01),
  method ="Broyden")\$x
[1] 8.023027e+06 1.416185e-02
> nleqslv(c(VE,SE), fnewton, control=list(btol=.01),
  method ="Newton")\$x
[1] 8.023027e+06 1.416185e-02
```

# Iteration

```
> D1<-function(V0,Z,r,sigmaV,T)
+ {(log(V0/Z) + (r + sigmaV^2/2)*T)/sigmaV/sqrt(T)}
> D2<-function(d1,sigmaV,T) {d1-sigmaV*sqrt(T)}
> f1<-function(Va)
+ {Va*pnorm(D1(Va,D,R,SA,1))-exp(-R)*D*pnorm(D2(D1(
  Va,D,R,SA,1),
+ SE,1))-VE}
> f2<-function(Sa) {VA/VE*pnorm(D1(VA,D,R,Sa,1))*Sa-
  SE}
> IT1<-VE; IT2<-SE; counter<-0
> while ( sqrt((SA-IT1)^2+(VA-IT2)^2)>0.1*(1+sqrt(
  IT1^2+IT2^2))
+ and counter<1000 )
> {SA<-IT2; IT1<-uniroot(f1,c(0,VE*100))\$root
+  VA<-IT1; IT2<-uniroot(f2,c(0,SE*100))\$root
+  counter<-counter+1}
```

# Reduction to one-dimensional case

```
> f <- function(x) {
+ VA = x[1]
+ SA = x[2]
+ d1 = (log(VE/D) + (R + SA^2/2)*T)/SA/sqrt(T)
+ d2 = d1 - SA*sqrt(T)
+ e1 = VE - (VA*pnorm(d1) - exp(-R*T)*D*pnorm(d2))
+ e2 = SE * VE - pnorm(d1)*SA*VA
+ return(e1^2 + e2^2)
+ }
> nlminb(c(VE,SE), f, lower=c(0, 0), upper=c(1E10, 1
  E3),)\$par
[1] 8.023027e+06 1.416185e-02
```

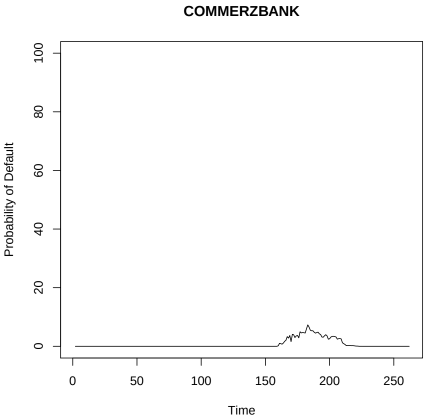


Figure 1: Default probability using Distance-to-Default

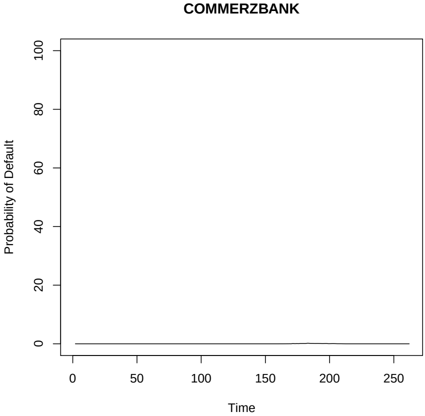


Figure 2: Default probability using Distance-to-Capital

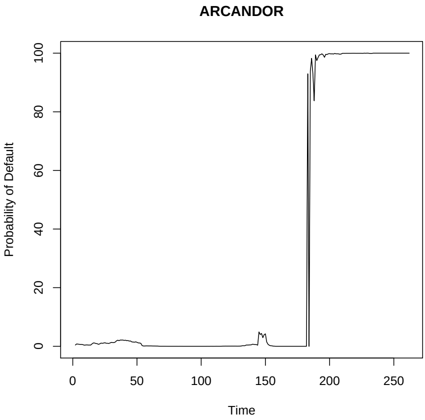


Figure 3: Default probability using Distance-to-Default

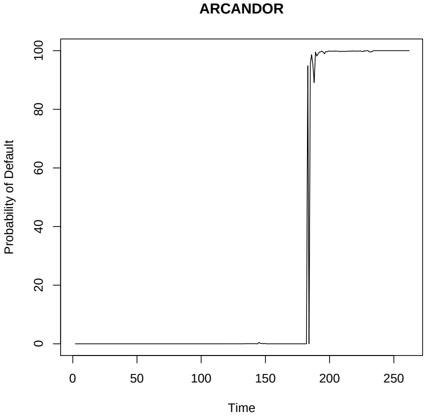


Figure 4: Default probability using Distance-to-Capital

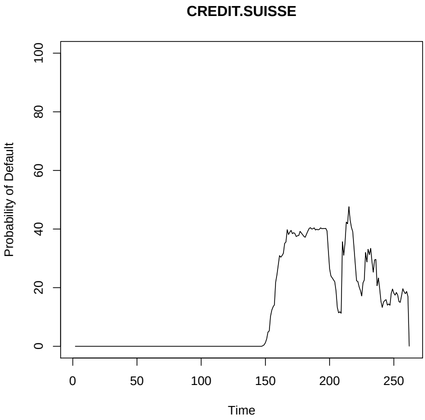


Figure 5: Default probability using Distance-to-Default

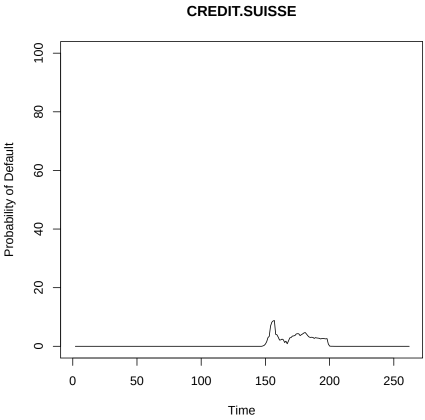


Figure 6: Default probability using Distance-to-Capital

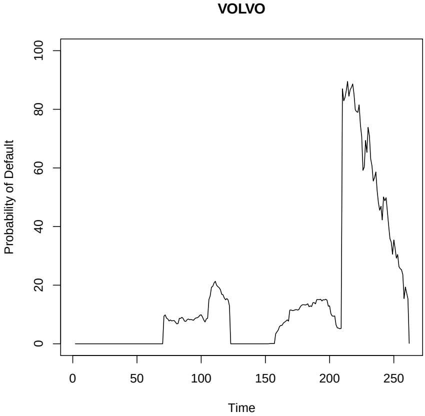


Figure 7: Default probability using Distance-to-Default

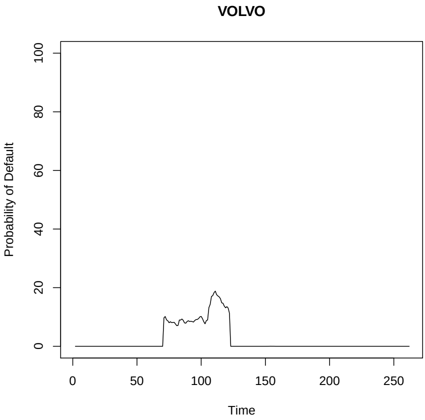


Figure 8: Default probability using Distance-to-Capital

# Conclusions:

- Some financial problems can be predicted 1 or even 1.5 year before.
- For financial companies Distance-to-Capital is more appropriate for calculating the probability of default.
- Model is more useful for rating than predicting.
- Most of financial companies had higher probability of default during the crisis.

# Further extensions:

- Estimation of volatility in different ways.
- Estimation of interest rate in different ways.
- Different frequencies of data.
- Comparison of US and European companies.
- Different time horizon.
- Impact of crisis on different industries.
- Impact of Basel II requirements.

# Data sources:

price and number of stocks
database: Datastream

Euribor
database: Datastream

# References and articles:


- Crosbie P., Bohn J. (2004): *Modelling default risk*, Published by Moody's KMV Company
- Bharath S.T., Shumway T. (2004): *Forecasting Default with the KMV-Merton Model*, University of Michigan
- Lu Y. (2008): *Default Forecasting in KMV*, Master thesis, Oxford University
- Larssen M., Magne A. (2010): *Predicting the default probability of companies in USA and EU during financial crisis*, Master thesis, Lund University